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Mathematics. — *"The force field of the non-Euclidean spaces with positive curvature"* by Mr. L. E. J. BROUWER. (Communicated by Prof. D. J. KORTEWEG).

(Communicated in the meeting of September 29, 1906).

D [1]). *The spherical Sp_n.*

I. The theorems under C § I and II hold invariably for the spherical and elliptical Sp_n's. But on account of the finiteness of these spaces we need not postulate a limiting field property for the following developments. We shall first consider the spherical spaces.

Firstly we remark for the general linevector distribution of the spherical Sp_n that the total sum of the divergency is 0; for the outgoing vectorcurrents out of the different space-elements destroy each other. This proves already that as elementary ${}^1_0X$ we can but take the field of a double point.

SCHERING (Göttinger Nachrichten 1873), and KILLING (Crelle's Journal, 1885) give as elementary gradient field the derivative of the potential function $\int_r^{1/2\pi} \frac{dr}{sin^{n-1} r} \equiv v_n(r)$. [2])

But the derivative of this field consists of two equal and opposite divergencies in two opposite points; and it is clear that an arbitrary integral of such fields always keeps equal and opposite divergencies in the opposite points, so it cannot furnish the general divergency-distribution limited only to a total divergency sum $= 0$.

II. If we apply for a spherical Sp_n the theorem of GREEN to the whole space (i. e. to the two halves, in which it is divided by an arbitrary closed Sp_{n-1}, together), doing this particularly for a scalar function φ which we presuppose to have nowhere divergency and a scalar function having only in two arbitrary points P_1 and P_2 equal and opposite divergencies and nowhere else (such functions we shall deduce in the following), we then find

$$\varphi_{P_1} - \varphi_{P_2} = 0,$$

i. o. w. φ is a constant, the points P_1 and P_2 being taken arbitrarily.

[1]) A, B and C refer to: "The force field of the non-Euclidean spaces with negative curvature". (See these Proceedings, June 30, 1906).

[2]) We put the space constant $= 1$, just as we did in the hyperbolic spaces.

So there is no ${}^{1}_{0}X$ possible with nowhere divergency, thus no ${}^{1}X$ having nowhere rotation and nowhere divergency, and from this ensues:

A linevector distribution in a spherical Sp_n is determined uniformly by its rotation and its divergency.

III. The general vector distribution in a spherical Sp_n must thus be obtainable again as an arbitrary integral of:

1. fields E_1, whose second derivative consists of two equal and opposite scalar values close to each other.

2. fields E_2, whose first derivative consists of planivectors distributed regularly in the points of a small ${}^{n-2}$sphere and perpendicular to that ${}^{n-2}$sphere.

At finite distance from their origin the fields E_1 and E_2 have an identical structure.

IV. For the spherical Sp_2 there exists a simple way to find the field E_1 namely conform representation by stereographic projection of a Euclidean plane with a doublepoint potential, which double point is situated in the tangential point of the sphere and the plane. If we introduce on both surfaces as coordinates the distance to the double point and the angle of the radiusvector with the doublepoint-axis — in the plane ϱ and φ, on the sphere r and φ — we have:

$$\tfrac{1}{2}\varrho = tan\ \tfrac{1}{2} r.$$

The potential in the plane: $\dfrac{cos\ \varphi}{\varrho}$ becomes on the sphere:

$$\tfrac{1}{2}\ cos\ \varphi\ cot\ \tfrac{1}{2} r.$$

This potential shows nothing particular in the centre of projection on the sphere, so it is really the potential to be found of a single double point, the field E_1. (If we place in the opposite point of the double point an other double point in such a way that the unequal poles correspond as opposite points, we find as potential $\tfrac{1}{2}\ cos\ \varphi\ (cot\ \tfrac{1}{2} r + tan\ \tfrac{1}{2} r) = \dfrac{cos\ \varphi}{sin\ r}$, which is the Schering potential of a double point).

V. Here too we can meanwhile break up the field of a double point into two fictitious "fields of a single agens point"; for this we have but to take $\int_r^{\pi} \tfrac{1}{2}\ cot\ \tfrac{1}{2} r\ dr = -l\ sin\ \tfrac{1}{2} r \equiv F(r)$; so that for an

arbitrary gradient distribution holds

$$ {}^1_0X = \overline{1}\int \frac{\overline{2}\,{}^1_0X}{2\pi} F(r)\, d\tau \ldots\ldots\ldots (I) $$

The "field of a single agens point" has however divergencies everywhere on the sphere.

VI. Out of the field E_1 we deduce in an analogous way as under B § VI the field E_2 of a rotation double point normal to the agens-doublepoint of the field E_1. As scalar value of the planivector potential we find there:

$$ \tfrac{1}{2} \sin\varphi \cot \tfrac{1}{2} r, $$

as we had to expect, completely dual to the scalar potential of the field E_1.

As fictitious force field of a unity-rotationelement we deduce out of this (in the manner of B § VI):

$$ \tfrac{1}{2} \cot \tfrac{1}{2} r, $$

directed normally to the radiusvector. For the rest this force field has rotation everywhere in Sp_3.

VII. Out of this we find (comp. under B § VII) for the scalar value of the planivector potential of a rotation-element:

$$ \int_r^\pi \tfrac{1}{2} \cot \tfrac{1}{2} r\, dr \equiv F(r), $$

so that for an arbitrary ${}^1_2X$:

$$ {}^1_2X = \overline{2}\int \frac{\overline{1}\,{}^1_2X}{2\pi} F(r)\, d\tau \ldots\ldots\ldots (II) $$

And an arbitrary vectorfield is the ∇ of a potential:

$$ \int \frac{\nabla X}{2\pi} F(r)\, d\tau. $$

E. The spherical Sp_3.

I. The purpose is in the first place to find E_1; we shall compose it of some singular potential functions with simple divergency distributions, and which are easy to construct.

Let us suppose a principal sphere B with poles P_1 and P_2, and on B a principal circle C with poles Q_1 and Q_2 determining on B meridian circles M cutting C in points H.

We can construct in the first place out of the SCHERING potential the potential of two double points, in P_1 and P_2, the positive poles of which are both directed towards Q_1 (so that in opposite points unequal poles correspond). Let us determine a point S of the hypersphere by the distance $PS = r$ and $\angle QPS = \varphi$ (where for P and Q the index 1 or 2 must be taken according to S lying with P_1 or with P_2 on the same side of B), then this potential (α) becomes

$$\pm \frac{\cos \varphi}{\sin^2 r},$$

where the sign + (—) must be taken for the half hyperspheres between P_1 (P_2) and B.

This field has no other divergency but that of the double points P_1 and P_2.

If we now reverse the sign of the potential in the half hypersphere on the side of P_2, we obtain the potential (β):

$$\frac{\cos \varphi}{\sin^2 r}.$$

The divergency of this consists in the first place of two double points, *one* directed in P_1 towards Q_1 and *one* directed in P_2 towards Q_2 (so that now in two opposite points equal poles correspond); and then of a magnetic scale (indeed a potential discontinuity) in sphere B varying in intensity according to $\cos \varphi$.

II. By the side of this we wish to find a potential, the divergency of which consists of only such a magnetic scale in sphere B with an intensity proportional to $\cos \varphi$. Now a field of a magnetic scale in B with an intensity varying according to an other zonal spherical harmonic, is easy to find. Let us namely take in each "meridian sphere" PQH as potential of a point S the angle $PHS = \frac{1}{2}\pi - \angle QHS$ (P and Q to be provided with indices in the way indicated above according to the place of S) $= \tan^{-1}\{\cos \varphi \tan r\}$, then we have such a potential: in the hypersphere it is a zonal spherical harmonic about PQ as axis; on the sphere B it has its only divergency in the shape of a magnetic scale, the intensity of which varies according to a zonal spherical harmonic with pole Q.

Let us now take in turns all the points of the sphere B as pole Q' of such a potential function, and let us integrate all those potentials over the solid angle about P each potential being multiplied by $\cos Q'Q$, then according to a wellknown theorem on spherical harmonics the integral is a zonal harmonic of form $\cos \varphi f(r)$, where

$$f(r) = \int \cos \varphi \,.\, \tan^{-1}\{\cos \varphi \tan r\}\, d\omega\,, \quad (d\omega \text{ representing the element}$$

of the solid angle about P), whilst this integral field has as only divergency a magnetic scale in B with intensity proportional to $\cos\varphi$.

Effecting the integration we obtain:

$$f(r) = 2\pi \int_0^{\pi} \sin\varphi \cos\varphi \tan^{-1}\{\cos\varphi \tan r\}\, d\varphi.$$

$$f(r) = 2\pi \left\{ -\cot r + \frac{r}{\sin^2 r} \right\},$$

and for the corresponding potential function (γ) we find:

$$2\pi \cos\varphi \left\{ -\cot r + \frac{r}{\sin^2 r} \right\}.$$

III. If we take the difference of the field (β) multiplied by $\frac{1}{2}$ and the field (γ) multiplied by $\frac{1}{2\pi^2}$ the magnetic scale in B disappears and we have left the field (δ):

$$\frac{\cos\varphi}{\pi} \left\{ \frac{\frac{\pi}{2} - r}{\sin^2 r} + \cot r \right\},$$

which field has as only divergency two double points in P_1 and P_2 of which in the opposite points equal poles correspond.

The sum of this field (δ) and the field (α) multiplied by $\frac{1}{2}$ must now give a field having as divergency a single double point with unity-moment in P_1, i. o. w. the field E_1.

We therefore find on the half hypersphere between P_1 and B:

$$\frac{1}{\pi} \cos\varphi \left\{ \frac{\pi - r}{\sin^2 r} + \cot r \right\}$$

and on the half hypersphere between P_2 and B:

$$\frac{1}{\pi} \cos\varphi \left\{ \frac{-r}{\sin^2 r} + \cot r \right\},$$

or if we define on both halves the coordinates r and φ according to P_1 and $P_1\, Q_1$ we obtain the following expression holding for both halves:

$$\frac{1}{\pi} \cos\varphi \left\{ \frac{\pi - r}{\sin^2 r} + \cot r \right\} \equiv \psi(r) \cos\varphi.$$

IV. To break up this field into two fictitious "fields of a single agens point" (having however divergency along the whole hypersphere) we take for the latter $\int_r^{\pi} \psi(r)\, dr \equiv F_1(r)$.

Then for an arbitrary gradient distribution holds:

$$ {}_0^1X = \overline{\backslash 1/} \int \frac{\overline{\backslash 2/}\ {}_0^1X}{4\pi} F_1(r)\, d\tau \ldots \ldots \quad (I) $$

V. The field E_1 of a circular current according to the equator plane in the origin, is identical outside the origin to the above field E_1; but now each force line is closed, and has a line integral of 4π along itself.

According to the method of A § IX we find of this field E_2 the planivector potential H in the meridian plane and independent of the azimuth.

We find when writing $\pi - r = \beta$:

$$ \Sigma = \frac{1}{\pi} \sin^2\varphi\,(1 + \beta \cot r)\, d\vartheta. $$

So

$$ H = \frac{1}{\pi} \sin\varphi \frac{1 + \beta \cot r}{\sin r}, $$

vanishing along all principal circles in the opposite point.

From which we deduce for the force of an element of current with unity-intensity in the origin directed according to the axis of the spherical system of coordinates:

$$ \frac{1}{\pi} \sin\varphi \frac{1 + \beta \cot r}{\sin r}, $$

directed normally to the meridianplane.

VI. From this we deduce as in A § XI a vector potential V of an element of current parallel to that element of current and a function of r only. For the scalar value U of that vector potential we have the differential equation:

$$ -\frac{\partial}{\partial r}\left\{ U \sin\varphi \sin r\, d\varphi \right\} dr - \frac{\partial}{\partial \varphi}\left\{ U \cos\varphi\, dr \right\} d\varphi = $$

$$ = \frac{1}{\pi} \sin\varphi \frac{1 + \beta \cot r}{\sin r} \,.\, dr \,.\, \sin r\, d\varphi. $$

Or:

$$ U - \frac{\partial}{\partial r}\left\{ U \sin r \right\} = \frac{1}{\pi}(1 + \beta \cot r), $$

of which the solution is

$$ U = \frac{c}{\cos^2 \frac{1}{2} r} + \frac{1}{\pi}\left\{ \frac{\frac{1}{4}\beta^2}{\cos^2 \frac{1}{2} r} - \frac{\beta}{\sin r} \right\}. $$

We choose $c = 0$, and we find as vector potential V of a unity-element of current:

$$\frac{1}{\pi}\left\{\frac{\frac{1}{4}\beta^2}{\cos^2\frac{1}{2}r} - \frac{\beta}{\sin r}\right\} \equiv F_2(r),$$

directed parallel to the element of current. The function $F_2(r)$ vanishes in the opposite point.

For an arbitrary flux now holds:

$$\tfrac{1}{2}X = \overline{\backslash 2 /}\int \frac{\overline{\backslash 1 /}\,\tfrac{1}{2}X}{4\pi} F_2(r)\,d\tau. \quad . \quad . \quad . \quad . \quad . \quad (II)$$

And finally the arbitrary vector field X is the ∇ of the potential:

$$\int \frac{\overline{\backslash 2 /}X}{4\pi} F_1(r)\,d\tau + \int \frac{\overline{\backslash 1 /}X}{4\pi} F_2(r)\,d\tau.$$

F. *The spherical Sp_n.*

I. To find the field E_1 we set to work in an analogous way as for the spherical Sp_3. The principal sphere B becomes here a $^{n-1}$sphere B; the principal circle C of the points H a principal $^{n-2}$sphere C of the points H.

For the potential (α) is found:

$$\pm\frac{\cos\varphi}{\sin^{n-1} r};$$

for the potential (β):

$$\frac{\cos\varphi}{\sin^{n-1} r};$$

this field (β) has in the sphere B a magnetic $^{n-1}$scale.

The potential (γ) is integrated out of fields $\tan^{-1}\{\cos\varphi\tan r\}$ according to $\cos\varphi$, the first zonal $^{n-1}$spherical harmonic on B. This integration furnishes when dw represents the element of the n-dimensional solid angle about P:

$$\cos\varphi\, f(r),$$

where:

$$f(r) = \int \cos\varphi\tan^{-1}\left\{\cos\varphi\tan r\right\} dw = k_{n-1}\int_0^\pi \sin^{n-2}\varphi\cos\varphi\tan^{-1}\left\{\cos\varphi\tan r\right\} d\varphi =$$

$$= \frac{k_{n-1}}{n-1}\int_0^\pi \sin^n\varphi\,\frac{\tan r\,d\varphi}{1+\tan^2 r\cos^2\varphi}$$

(k_n defined as under C § III).

Putting under the sign of the integral a factor $\sin^2\varphi \tan^2 r$ outside the brackets and, by regarding that factor as $\frac{1}{\cos^2 r} - (1 + \cos^2\varphi \tan^2 r)$, writing the integral as sum of two integrals to the former of which the same division in two is applied, etc., we find, if we write $\int_0^\pi \sin^h r\, dr = S_h$:

$$\frac{(n-1) f(r)}{k_{n-1}} \sin^{n-1} r = -\sin^{n-2} r \cos r\, S_{n-2} - \sin^{n-4} r \cos r\, S_{n-4} \ldots .$$

$$\ldots . - \sin^2 r \cos r\, S_2 + \pi (1 - \cos r)$$

(for n even)

$$= -\sin^{n-2} r \cos r\, S_{n-2} - \sin^{n-4} r \cos r\, S_{n-4} \ldots .$$

$$\ldots . - \sin r \cos r\, S_1 + 2r$$

(for n odd)

$$= \pi \cdot \frac{(n-1)(n-3)\ldots .}{(n-2)(n-4)\ldots .} \int_0^r \sin^{n-1} r\, dr = (n-1) S_{n-2} \int_0^r \sin^{n-1} r\, dr,$$

(for n even)

$$= 2 \cdot \frac{(n-1)(n-3)\ldots .}{(n-2)(n-4)\ldots .} \int_0^r \sin^{n-1} r\, dr = (n-1) S_{n-2} \int_0^r \sin^{n-1} r\, dr.$$

(for n odd)

If we write ξ_n for $2 \cdot \pi \cdot 2 \cdot \pi \cdot 2 \ldots .$, to n factors, we have

$$k_n = \frac{\xi_n}{(n-2)(n-4)\ldots .}, \text{ and } \frac{k_{n+1}}{k_n} = S_{n-1}.$$

Therefore:

$$f(r) \sin^{n-1} r = k_n \int_0^r \sin^{n-1} r\, dr,$$

and the potential (γ) becomes:

$$k_n \frac{\cos\varphi}{\sin^{n-1} r} \int_0^r \sin^{n-1} r\, dr.$$

II. We find the field (δ) by taking difference of field (β) multiplied by $\frac{1}{2}$ and field (γ) by $\frac{1}{k_n S_{n-1}} = \frac{1}{k_{n+1}}$, i. e.

$$\frac{\cos\varphi}{\sin^{n-1}r}\cdot\frac{\frac{1}{2}S_{n-1}-\int_0^r \sin^{n-1}r\,dr}{S_{n-1}}\equiv\frac{\cos\varphi}{\sin^{n-1}r}\cdot\frac{\int_r^{\frac{1}{2}\pi}\sin^{n-1}r\,dr}{S_{n-1}}.$$

This field has as only divergency two double points, in P_1 and P_2, of which equal poles correspond in the opposite points. The field E_1 is then obtained by adding to it the field (α) multiplied by $\frac{1}{2}$. We find on the half $n$sphere between P_1 and B:

$$\frac{1}{S_{n-1}}\cdot\frac{\cos\varphi}{\sin^{n-1}r}\int_r^{\pi}\sin^{n-1}r\,dr.$$

On the half $n$sphere between P_2 and B:

$$-\frac{1}{S_{n-1}}\cdot\frac{\cos\varphi}{\sin^{n-1}r}\int_0^{r}\sin^{n-1}r\,dr.$$

Or, if we define on both halves the coordinates r and φ according to P_1 and P_1Q_1, we arrive at the expression holding for both halves:

$$\frac{1}{S_{n-1}}\cdot\frac{\cos\varphi}{\sin^{n-1}r}\cdot\int_r^{\pi}\sin^{n-1}r\,dr\equiv\psi_n(r.)\cos\varphi.$$

III. For the potential of the fictitious "field of a single agens point" we find:

$$\int_r^{\pi}\psi_n(r)\,dr\equiv F_1(r).$$

And for the arbitrary gradient distribution holds:

$${}_0^1X=\overline{\backslash 1/}\int\frac{\overline{\backslash 2/}\,{}_0^1X}{k_n}F_1(r)\,d\tau\quad\ldots\quad(I)$$

Of the divergency distribution of $F_1(r)$ in points of a general position we know that, taken for two completely arbitrary centra (fictitious agens points) with opposite sign and then summed up, it furnishes 0: so on one side that distribution is independent of the position of the centre and on the other side it lies geometrically equivalent with respect to all points; so it is a constant. But if the function $F_1(r)$ has constant divergency in points of general position it satisfies a differential equation putting the divergency constant. In this is therefore a second means to determine the function F_1 and out of this the field E_1.

The differential equation becomes:

$$\frac{d}{dr}\left\{\sin^{n-1}r \cdot \frac{dF_1}{dr}\right\} = c \sin^{n-1}r \quad . \quad . \quad . \quad . \quad . \quad (H)$$

$$\sin^{n-1}r \cdot \frac{dF_1}{dr} = c \int \sin^{n-1}r \, dr.$$

$$\frac{dF_1}{dr} = c \cdot \frac{\int \sin^{n-1}r \, dr}{\sin^{n-1}r}.$$

If the field E_1 is to be composed out of the function $F_1(r)$ then the opposite point of the centre may not have a finite outgoing vector current; we therefore put $\int \sin^{n-1}r \, dr = 0$, so that we get

$$\frac{dF_1}{dr} = -\frac{c}{\sin^{n-1}r} \int_r^{\pi} \sin^{n-1}r \, dr,$$

which corresponds to the above result.

IV. The field E_2 of a small vortex $^{n-2}$sphere according to Sp_{n-1}, perpendicular to the axis of the just considered double point, is identical to that field E_1 outside the origin; but now each force line is closed and has a line integral k_n along itself.

According to the method of C § VII we shall find of this field E_2 the planivector potential H, lying in the meridian plane and dependent only on r and φ; so that it is a ${}^2_1X$. We find:

$$dh = c\varepsilon \sin^{n-2}r \sin^{n-2}\varphi.$$

Force in r-direction:

$$(n-1)\cos\varphi \left\{\frac{1}{(n-1)S_{n-1}} + \frac{\cot r}{S_{n-1}} \cdot \frac{\int_r^{\pi} \sin^{n-1}r \, dr}{\sin^{n-1}r}\right\} \equiv (n-1)\cos\varphi \cdot \omega_n(r.)$$

$$\Sigma = \int_0^{\varphi} (n-1)\cos\varphi \, \omega_n(r) \cdot c\varepsilon \sin^{n-2}r \sin^{n-2}\varphi \cdot \sin r \, d\varphi =$$

$$= \omega_n r \cdot c\varepsilon \sin^{n-1}r \sin^{n-1}\varphi.$$

$$H = \frac{\Sigma}{dh} = \omega_n(r) \sin r \sin\varphi \equiv \chi_n(r) \sin\varphi.$$

From this ensues for the force of a plane vortex element with unity-intensity in the origin:

$$\chi_n(r) \sin\varphi,$$

directed parallel to the acting vortex element and projecting itself on that plane according to the tangent to a concentric circle; whilst φ is the angle of the radiusvector with the Sp_{n-2} perpendicular to the vortex element.

V. In the same way as in C § IX we deduce from this the planivector potential V of a vortex element directed everywhere parallel to the vortex element and of which the scalar value is a function of r only. That scalar value U of that vector potential is here determined by the differential equation:

$$-\frac{\partial}{\partial\varphi}\left\{U\cos\varphi\,.\,dr\,.\,c\varepsilon\sin^{n-3}r\cos^{n-3}\varphi\right\}d\varphi -$$

$$-\frac{\partial}{\partial r}\left\{U\sin\varphi\,.\,\sin r\,d\varphi\,.\,c\varepsilon\sin^{n-3}r\cos^{n-3}\varphi\right\}dr =$$

$$= \chi_n(r)\sin\varphi\,.\,\sin r\,d\varphi\,.\,dr\,.\,c\varepsilon\sin^{n-3}r\cos^{n-3}\varphi.$$

$$(n-2)\,U - \frac{dU}{dr}\sin r - (n-2)\,U\cos r = \chi_n(r)\sin r.$$

$$\frac{dU}{dr} - (n-2)\,U\,tg\,\tfrac{1}{2}r = -\chi_n(r).$$

$$U = \frac{1}{\cos^{2(n-2)}\frac{1}{2}r}\cdot\int_r^{\pi}\cos^{2(n-2)}\tfrac{1}{2}r\cdot\chi_n(r)\,dr,$$

a function vanishing in the opposite point, which we put $= F_2(r)$. We then find for an arbitrary flux:

$${}^1_2X = \overline{\langle 2\rangle}\int\frac{\overline{\langle 1\rangle}\,{}^1_2X}{k_n}F_2(r)\,d\tau \quad . \quad . \quad . \quad . \quad . \quad . \quad (II)$$

And taking an arbitrary vector field to be caused by its two derivatives (the magnets and the vortex systems) propagating themselves through space as a potential according to a function of the distance vanishing in the opposite point, we find:

$$X = \nabla\left\{\int\frac{\overline{\langle 2\rangle}X}{k_n}F_1(r)\,d\tau + \int\frac{\overline{\langle 1\rangle}X}{k_n}F_2(r)\,d\tau\right\}.$$

G. *The Elliptic Sp_n.*

Also for the elliptic Sp_n the derivative of an arbitrary linevector distribution is an integral of elementary vortex systems Vo_y and Vo_z, which are respectively the first and the second derivative of

an isolated line vector. For elementary ${}^1_0X$ we shall thus have to put the field of a divergency double point.

The Schering elementary potential $\int_r^{1/2\pi} \frac{dr}{\sin^{n-1} r} \equiv v_n(r)$ is here a plurivalent function (comp. KLEIN, Vorlesungen über Nicht-Euklidische Geometrie II, p. 208, 209); it must thus be regarded as senseless.

II. The unilateral elliptic Sp_n is enclosed by a plane Sp_{n-1}, regarded twice with opposite normal direction, as a bilateral singly connected Sp_n-segment by a bilateral closed Sp_{n-1}. If we apply to the Sp_n enclosed in this way the theorem of GREEN for a scalar function φ having nowhere divergency, and for one having in two arbitrary points P_1 and P_2 equal and opposite divergencies and fartheron nowhere (such a function will prove to exist in the following), we shall find:

$$\varphi_{P_2} - \varphi_{P_1} = 0,$$

i. o. w. φ is a constant, the points P_1 and P_2 being arbitrarily chosen.

So no ${}^1_0X$ is possible having nowhere divergency, so no ${}^1X$ having nowhere rotation and nowhere divergency; and from this ensues:

A linevector distribution in an elliptical Sp_n is uniformly determined by its rotation and its divergency.

III. So we consider:

1. the field E_1, with as second derivative two equal and opposite scalar values quite close together.

2. the field E_2 with as first derivative planivectors regularly distributed in the points of a small $^{n-2}$sphere and perpendicular to that small $^{n-2}$sphere.

At finite distance from their origin the fields E_1 and E_2 are of identical structure.

IV. To find the potential of the field E_1 we shall represent it uni-bivalently on the spherical Sp_n; the representation will have as divergency two doublepoints in opposite points, where equal poles correspond as opposite points; it will thus be the field (σ), deduced under F § II, multiplied by 2:

$$\frac{\cos\varphi}{\sin^{n-1}r}\cdot\frac{\int_r^{1/2\pi}\sin^{n-1}r\,dr}{\frac{1}{2}S_{n-1}}\equiv\lambda_n(r)\cos\varphi.$$

In the field corresponding to this in the elliptic space, all force lines move from the positive to the negative pole of the double point; a part cuts the pole Sp_{n-1} of the origin: these force lines are unilateral in the meridian plane; the remaining do not cut it; these are bilateral in the meridian plane.

The two boundary force lines forming together a double point in the pole Sp_{n-1}, have the equation:

$$\sin^{n-1}\varphi\left\{\sin^{n-1}r+(n-1)\cot r\int_r^{1/2\pi}\sin^{n-1}r\,dr\right\}=\pm 1.$$

The Sp_{n-1} of zero potential consists of the pole Sp_{n-1} and the equator Sp_{n-1} of the double point; its line of intersection with the meridian plane has a double point in the force lines doublepoint. All potential curves in the meridian plane are bilateral.

V. For the fictitious "field of a single agens point" the potential is $\int\lambda_n(r)\,dr$. It is rational to let it become 0 in the pole Sp_{n-1}; so we find:

$$\int_r^{1/2\pi}\lambda_n(r)\,dr\equiv F_1(r),$$

and for the arbitrary gradient distribution holds:

$${}^1_0X=\overline{\backslash 1/}\int\frac{\overline{\backslash 2/}\,{}^1_0X}{k_n}F_1(r)\,d\tau\quad\ldots\ldots\quad(I)$$

We could also have found $F_1(r)$ out of the differential equation (H) of F § III, which it must satisfy on the same grounds as have been asserted there. For the elliptic Sp_n also we find:

$$\frac{dF_1}{dr}=c\cdot\frac{\int\sin^{n-1}r\,dr}{\sin^{n-1}r}.$$

But here in the pole Sp_{n-1}, lying symmetrically with respect to the centre of the field, the force, thus $\int\sin^{n-1}r\,dr$ must be 0; so that we find:

$$\frac{dF_1}{dr} = -\frac{c}{\sin^{n-1} r} \int_r^{1/2\pi} \sin^{n-1} r \, dr.$$

VI. In the usual way we deduce the ${}^2_1X$, which is planivector potential of the field E_1.

$$dh = c\varepsilon \sin^{n-2} r \sin^{n-2} \varphi.$$

Force in r-direction:

$$(n-1)\cos\varphi \left\{ \frac{2}{(n-1)S_{n-1}} + \frac{2\cot r}{S_{n-1}} \cdot \frac{\int_r^{1/2\pi} \sin^{n-1} r \, dr}{\sin^{n-1} r} \right\} \equiv (n-1)\cos\varphi \,.\, \mu_n(r).$$

$$\Sigma = \int_0^{\varphi} (n-1)\cos\varphi \,.\, \mu_n(r) \,.\, c\varepsilon \sin^{n-2} r \sin^{n-2}\varphi \,.\, \sin r \, d\varphi =$$

$$= \mu_n(r) \,.\, c\varepsilon \sin^{n-1} r \sin^{n-1}\varphi.$$

$$H = \frac{\Sigma}{dh} = \mu_n(r) \sin r \sin\varphi \equiv \varkappa_n(r) \sin\varphi.$$

From which ensues for the force of a plane vortex element with unity-intensity in the origin:

$$\varkappa_n(r) \sin\varphi,$$

directed parallel to the acting vortex element and projecting itself on its plane according to the tangent to a concentric circle; φ is here the angle of the radiusvector with the Sp_{n-2} perpendicular to the vortex element.

VII. Here too a planivector potential of a vortex element can be deduced, but we cannot speak of a direction propagated parallel to itself, that direction not being uniformly determined in elliptic space; after a circuit along a straight line it is transferred into the symmetrical position with respect to the normal plane on the straight line.

But we can obtain a vector potential determined uniformly, by taking that of two antipodic vortex elements in the spherical Sp_n (in their ²sphere the two indicatrices are then oppositely directed).

The vector potential in a point of the elliptic Sp_n then lies in the space through that point and the vortex element; if we regard the plane of the element as equator plane in that space then the planivector potential V is normal to the meridian plane: it consists of:

1. a component U_1 normal to the radiusvector, according to the formula:

$$\frac{U_1}{\cos\varphi} = \frac{1}{\cos^{2(n-2)}\frac{1}{2}r}\int_r^{\pi}\cos^{2(n-2)}\tfrac{1}{2}r\,\chi_n(r)\,dr + \\ + \frac{1}{\sin^{2(n-2)}\frac{1}{2}r}\int_{\pi-r}^{\pi}\cos^{2(n-2)}\tfrac{1}{2}r\,.\,\chi_n(r)\,dr.$$

2. a component U_2 through the radiusvector, according to the formula:

$$\frac{U_2}{\sin\varphi} = \frac{1}{\cos^{2(n-2)}\frac{1}{2}r}\int_r^{\pi}\cos^{2(n-2)}\tfrac{1}{2}r\,\chi_n(r)\,dr - \\ - \frac{1}{\sin^{2(n-2)}\frac{1}{2}r}\int_{\pi-r}^{\pi}\cos^{2(n-2)}\tfrac{1}{2}r\,.\,\chi_n(r)\,dr.$$

If we regard this planivector potential as function of the vortex element and the coordinates with respect to the vortex element and represent that function by G_2, then

$${}^1_2X = \overline{\vee^2}\int\frac{G_2\left\{\overline{\vee^1}\,{}^1_2X,\,r,\,\varphi\right\}}{k_n}\,d\tau. \quad . \quad . \quad . \quad . \quad (II)$$

holds for an arbitrary flux in the elliptic Sp_n.

And regarding an arbitrary vector field as caused by the two derivatives (the magnets and the vortex systems) propagating themselves through the space to a potential, we write:

$$X = \nabla\int\frac{\overline{\vee^2}\,X\,.\,F_1(r) + G_2(\overline{\vee^1}\,X,\,r,\,\varphi)}{k_n}\,d\tau.$$

VIII. In particular for the elliptic Sp_3 the results are:

Potential of an agens double point:

$$\frac{\cos\varphi}{\sin^2 r}\cdot\frac{\int_r^{\frac{1}{2}\pi}\sin^2 r\,dr}{\frac{1}{2}S_2} = \frac{2\cos\varphi}{\pi}\cdot\left\{\frac{(\frac{1}{2}\pi - r)}{\sin^2 r} + \cot r\right\},$$

or if we put $\frac{1}{2}\pi - r = \gamma$:

$$\frac{2\cos\varphi}{\pi}\cdot\left\{\frac{\gamma}{\sin^2 r} + \cot r\right\}.$$

Equation of the boundary lines of force:

$$\sin^2\varphi\,(1+\gamma\cot r) = \pm 1.$$

Potential of a single agens point:

$$-\frac{2}{\pi}\cdot\gamma\cdot\cot r.$$

Vector potential of an elementary circular current:

$$\frac{2}{\pi}\sin\varphi\cdot\frac{1+\gamma\cot r}{\sin r}.$$

So also force of an element of current:

$$\frac{2}{\pi}\sin\varphi\cdot\frac{1+\gamma\cot r}{\sin r}.$$

Linevector potential of an element of current:

according to the radiusvector: $\frac{\cos\varphi}{\pi}\left\{\frac{\frac{1}{4}\beta^2}{\cos^2\frac{1}{2}r}-\frac{\pi}{\sin r}+\frac{\frac{1}{4}r^2}{\sin^2\frac{1}{2}r}\right\}$.

normal to the radiusvector: $\frac{\sin\varphi}{\pi}\left\{\frac{\frac{1}{4}\beta^2}{\cos^2\frac{1}{2}r}+\frac{2r-\pi}{\sin r}-\frac{\frac{1}{4}r^2}{\sin^2\frac{1}{2}r}\right\}$.

IX. For the elliptic plane we find:

Potential of an agens double point:

$$\cos\varphi\cot r.$$

Equation of the boundary lines of force:

$$\sin\varphi = \pm\sin r, \quad \text{or} \quad \varphi = \begin{cases} r \\ \pi - r \end{cases}.$$

Potential of a single agens point:

$$-l\sin r.$$

Scalar value of the planivector potential of a double point of rotation:

$$\frac{\sin\varphi}{\sin r}.$$

Thus also force of a rotation element:

$$\frac{\sin\varphi}{\sin r}.$$

Planivector potential of a rotation element:

$$l\cot\tfrac{1}{2}r.$$

We notice that the duality of both potentials and both derivatives existing for the spherical Sp_2, has disappeared again in these results. The reason of this is that for the representation on the sphere a divergency in the elliptic plane becomes two equal divergencies in opposite points with equal signs; a rotation two equal rotations in opposite points with different signs; for the latter we do not find the analogous potential as for the former; the latter can be found here according to the Schering potential formula.

With this is connected immediately that in the elliptic plane the field of a single rotation (in contrast to that of a single divergency) has as such possibility of existence, so it can be regarded as unity

of field. That field consists of forces touching concentric circles and great $\frac{1}{\sin r}$.

Postscript. In the formula for vector fields in hyperbolic spaces:

$$Pot.\ X = \int \frac{\overline{\backslash 2/}\ X}{k_n} F_1(r)\, d\tau + \int \frac{\overline{\backslash 1/}\ X}{k_n} F_2(r)\, d\tau$$

nothing for the moment results from the deduction but that to $\overline{\backslash 2/}\ X$ and $\overline{\backslash 1/}\ X$ also must be counted the contributions furnished by infinity. From the field property ensues, however, immediately that the effect of these contributions disappears in finite, so that under the integral sign we have but to read $\overline{\backslash 2/}\ X$ and $\overline{\backslash 1/}\ X$ in finite.

For the $\overline{\backslash 1/}$ at infinity pro surface-unity of the infinitely great sphere is $<$ order e^{-r}; the potential-effect of this in finite becomes $<$ order $re^{-(n-2)r} \times e^{-r} = re^{-(n-1)r}$; so the force-effect $<$ order $e^{-(n-1)r}$; so the force-effect of the entire infinitely great spherical surface is infinitesimal.

And the $\overline{\backslash 2/}$ at infinity pro surface-unity is $<$ order $\frac{1}{r}$; it furnishes a potential-effect in finite $<$ order $e^{-(n-1)r} \cdot \frac{1}{r}$, thus a force-effect $<$ order $e^{-(n-1)r} \cdot \frac{1}{r^2}$; so the force-effect, caused by the infinite, remains $<$ order $\frac{1}{r^2}$.

The reasoning does not hold for the force field of the hyperbolical Sp_2 in the second interpretation (see under B § VIII), but it is in the nature of that interpretation itself that the derivatives at infinity are indicated as such, therefore also counted.

Meteorology. — "*On Magnetic Disturbances as recorded at Batavia.*" By Dr. W. VAN BEMMELEN.

(Communicated in the meeting of September 29, 1906).

Some months ago Mr. MAUNDER of the Greenwich-Observatory addressed a request to the Batavia Observatory to provide him with a list of magnetic storms recorded at Batavia with a view of testing his results as to the influence of the synodic rotation of the sun to the occurrence of disturbances.

Mr. MAUNDER concludes from an inspection of the disturbances

recorded at Greenwich (and also at Toronto) that they show a tendency to recur after a synodic rotation of the sun and that sometimes even two and more returns occur. His conclusion is:

"Our magnetic disturbances have their origin in the sun. The solar "action which gives rise to them does not act equally in all directions, "but along narrow, well defined streams, not necessarily truly radial. "These streams arise from active areas of limited extent. These active "areas are not only the source of our magnetic disturbances, but "are also the seats of the formation of sun-spots."

As soon as I could find the necessary leisure I prepared a list for the period 1880—1899, containing 1149 disturbances and immediately after made some statistical calculations based on them.

A discussion of such statistical results is always better made by the author of the list, than by another person for whom it is impossible to consult the original sheets.

Though intending to publish the list, statistics and some reproductions in full, I wish to give a preliminary account of my results, because these questions are now of actual importance.

Rules followed in preparing the list.

An exact definition of what is understood by the expression "magnetic storm" has never been given; certain features however are characteristic to it, viz:

1. The sudden commencement.
2. The postturbation.
3. The increased agitation.

Concerning the second, which I called the postturbation [1]), the well known fact may be remarked, that during a storm the *mean* level of the components of the force changes, till a maximum digression is reached, and afterwards returns slowly to its old value.

In 1895 I called attention to this phenomenon and investigated its distribution over the earth.

This research enabled me to give the following description of the postturbation.

During a magnetic storm a force appears contrary to the earth's ordinary magnetic force, with this difference, that its horizontal component is directed along the meridians of the regular part of the earth's magnetism, consequently not pointing to the magnetic pole, but to the mean magnetic axis of the earth.

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[1]) Cf. Meteorologische Zeitschrift 1895, p. 321. Terrestrisch Magnetisme I p. 95, II 115, V 123, VIII 153.

In accordance with this description, during the earlier part of a storm the horizontal force diminishes, the vertical force increases, and during the latter part these forces resume slowly their original values. The characteristic features sub 1 and 2 either do not necessarily attend every storm, or if so, they do not show themselves clearly enough to enable us to decide definitely whether a succession of waves in a curve must be considered as a storm or not.

On the contrary the increased agitation is an essential feature and has therefore been adopted by me as a criterion.

Unfortunately it is impossible to establish the lowest level above which the never absent agitation may be called a storm, because the agitation is not only determined by the amplitude of the waves, but also by their steepness and frequency.

To eliminate as much as possible the bad consequences which necessarily attend a personal judgment, the list has been prepared:

1. by one person;
2. in as short a time as possible;
3. from the aspect of the curves for one component only (in casu the horizontal intensity, which in Batavia is most liable to disturbance);
4. for a period with nearly constant scale-value of the curves (1 mm. $= \pm$ 0,00005 C.G.S.);

For each storm has been noted:

1. the hour of commencement;
2. „ „ „ expiration;
3. „ „ „ maximum;
4. the intensity.

Mr. MAUNDER calls a storm with a sudden start an S-storm; analogously I will call one with a gradual beginning a G-storm. In the case of a sudden impulse the time of beginning is given to the tenth of an hour; in that of gradual increase of agitation only by entire hours.

The hour of beginning of a G-storm is not easy to fix. I have chosen for it the time of the very beginning of the increased agitation, and not the moment in which the agitation begins to show an unmistakable disturbance character.

Afterwards it became clear I had shown a decided preference for the even hours, which may be accounted for by the fact that only the even hours are marked on the diagrams.

To eliminate this discordance I have added the numbers of G-storms

commencing at the odd hours for one half to the preceding and for the other half to the following hour.

Because a storm as a rule expires gradually, it is often impossible to give the exact moment it is past. If doubtful I have always taken the longest time for its duration; hence many days following a great storm are reckoned as being disturbed, which otherwise would have passed as undisturbed.

For the time of the maximum I have taken the moment of maximum agitation, which does not always correspond with the hour of maximum postturbation.

I believe the hour at which the mean H-force reaches its lowest level is a better time-measure for the storm-maximum, but to determine it a large amount of measuring and calculating is required, the change in level being often entirely hidden by the ordinary solar-diurnal variation.

The intensity of the storm has been given after a scale of four degrees: 1 = small; 2 = moderate; 3 = active; 4 = very active.

It is not possible to give a definition of this scale of intensity in words, the reproduction of typical cases would be required for this.

Hourly distribution of the beginning of storms.

It is a known fact, that the starting impulse is felt simultaneously all over the earth. The Greenwich and Batavia lists furnished me with 53 cases of corresponding impulses, which, if the simultaneity is perfect, must enable us to derive the difference in longitude of the two observatories.

I find in 6 cases 7^h12^m
„ 19 „ 7 0
„ 28 „ 7 6

Mean $7^h\ 7^m15^s$

True difference $7^h7^m19^s$.

It it very remarkable indeed to derive so large a difference of longitude with an error of 4 seconds only, from 53 cases measured roughly to 0.1 hour.

The simultaneity should involve an equal hourly distribution if every *S*-impulse were felt over the whole earth. As this is not the case, which is proved by the lists of Greenwich and Batavia, it is easy to understand that the Batavia-impulses show indeed an unequal hourly distribution. We find them more frequent at 6^h and 10^h a. m. and 7^h p. m.

Hourly distribution of *S*-impulses.

Hour	Number in %	Hour	Number in %
0 a. m.	4.1	noon 12	4.7
1	2.5	13	5.0
2	3.0	14	3.3
3	2.2	15	3.9
4	4.1	16	4.4
5	3.9	17	3.6
6	6.3	18	4.1
7	4.5	19	5.5
8	5.1	20	3.6
9	5.8	21	3.9
10	6.1	22	3.6
11	5.0	23	3.3

This same distribution we find again in the case of the *G*-storms, but much more pronounced; a principal maximum at about 8^h a. m., and a secondary one at 6^h p. m.

Accordingly the hour of commencement of the *G*-disturbances is dependent upon the position of the station with respect to the sun, and it seems, that the hours most appropriate for the development of a *G*-disturbance also favour the development of an *S*-impulse.

Hourly distribution of the commencement of G-storms (in %).

Hour	0	2	4	6	8	10	noon 12	14	16	18	20	22
Intensity: 1	6.0	6.0	5.1	6.7	18.7	17.9	7.4	4.7	6.0	8.0	6.5	7.1
» 2	4.5	4.9	4.2	5.7	20.8	16.4	7.3	5.6	5.4	9.2	8.4	7.6
» 3 and 4	7.1	3.5	4.3	7.5	18.5	13.4	3.9	5.5	5.1	9.1	11.8	10.2
All	5.4	5.1	4.6	6.3	19.7	16.5	6.8	5.2	5.6	8.7	8.3	7.8

Hourly distribrution of the maximum (in %).

	Hour	0	2	4	6	8	10	noon 12	14	16	18	20	22
S-storms.	Intensity 1	11.5	6.7	5.5	1.8	4.7	12.8	12.6	5.3	6.1	6.1	10.9	16.2
	» 2	16.9	9.1	5.9	1.2	1.4	7.3	7.6	3.9	7.6	10.8	13.5	15.2
	» 3 and 4	11.2	4.0	2.4	1.6	0.8	4.0	7.2	6.8	12.0	13.6	17.2	19.2
	All	14.1	7.4	4.9	1.5	2.4	8.6	9.2	4.8	7.8	9.7	13.3	16.3
G-storms.	Intensity 1	12.3	16.7	10.9	5.8	4.3	13.8	5.8	2.9	2.9	5.1	10.1	9.4
	» 2	11.3	7.7	3.6	5.6	3.3	8.5	11.3	8.5	6.1	5.2	14.1	14.9
	» 3 and 4	12.2	9.3	5.8	3.2	3.5	7.7	9.0	6.4	8.0	7.7	10.3	17.0
	All	11.9	10.2	6.0	4.6	3.6	9.2	9.2	6.5	6.3	6.3	11.6	14.8

These hourly numbers show for each intensity, and for both kinds of storms the same, strongly marked distribution over the hours of the day.

Thus the development of agitation during a storm is dependent on the position of the sun relatively to the station in a manner which is the same for S- and G-storms.

The period has a principal maximum at 10^h p. m. and a secondary one at noon; and being compared to the diurnal periodicity of the commencement of G-storms, it is evident, that: *On the hours when the chance for a maximum-agitation begins to increase, we may expect most storms to take a start.*

Hence we may come to the following supposition.

The susceptibility of the earth's magnetic field to magnetic agitation is liable to a diurnal and semidiurnal periodicity. Whatever may be the origin of the increase of agitation, sudden or gradual, this periodicity remains the same.

This was the same thing, that was revealed to me by the inspection of the hundreds of curves in preparing the list.

The agitation rises at about 8^h a. m. after some hours of great calm and reaches a maximum at about noon. A second period of calm, less quiet however than in the early morning, is reached in the afternoon, and a second rise follows till a maximum is attained shortly before midnight.

The day-waves however are smaller and shorter, the night-waves larger and longer and also more regular in shape. These regular night-waves are often restricted to one large wave, very suitable for the study of these waves.

Hourly distribution of the end of the storm.

Hour	Number of cases	Hour	Number of cases
0 a. m.	163	12	66
2	172	14	60
4	204	16	43
6	140	18	50
8	60	20	46
10	36	22	49

Quite in agreement with the above mentioned conclusions, the curve representing the diurnal periodicity of the final-hour is nearly the reverse of that for the maximum.

Evidently the hour 0 (the end of the day) has been strongly favoured.

Resuming we may according to the Batavia disturbance-record draw the following conclusions:

1st. *the origin of* S-*storms is cosmical*;

2nd. *the origin of* G-*storms may be also cosmical, but the commencement is dependent on the local hour;*

3rd. *the development of all storms, concerning the agitation, is in the same way dependent on the local hour.*

Storms and sunspots.

In the following table the year has been reckoned from April 1st till April 1st of the following year, with the exception of 1882, the diagrams for the months Dec. '82, Jan.—March '83 missing. For 1880—'83 the yearly numbers have been increased in proportion to the number of missing record days.

Numbers in %.

Year	Sunspot number	Intensity							
		1		2		3 and 4		All	
		S	G	S	G	S	G	S	G
1880/81	37.5	2.7	4.2	1.5	6.6	1.2	2.3	1.6	5.1
81/82	56.9	1.4	6.7	0.0	5.9	1.2	2.3	0.8	5.6
82	70.8	1.4	2.8	5.3	6.6	13.0	6.1	7.9	5.2
83/84	68.8	6.8	7.8	7.5	5.4	6.9	5.3	7.1	6.2
84/85	59.5	2.7	4.6	9.8	6.4	9.3	6.1	8.2	5.7
85/86	45.7	4.1	1.4	9.8	4.1	10.6	6.9	9.0	3.6
86/87	19.6	0.0	1.1	2.3	5.9	1.9	7.6	1.6	4.5
87/88	11.6	2.7	7.8	3.0	4.3	3.7	4.6	3.3	5.6
88/89	6.4	5.4	5.7	4.5	4.1	2.5	3.1	3.8	4.5
89/90	5.9	10.8	4.9	5.3	3.3	1.2	4.6	4.6	4.1
90/91	13.0	13.5	8.1	2.3	3.8	0.6	3.1	3.8	5.2
91/92	47.4	6.8	4.6	5.3	5.1	6.9	8.4	4.7	5.5
92/93	74.5	9.5	6.4	8.3	3.3	12.4	4.6	10.3	4.6
93/94	85.2	9.5	6.0	9.0	3.8	11.8	5.3	10.3	4.8
94/95	74.2	5.4	4.6	8.3	7.7	3.7	6.9	5.7	6.5
95/96	57.4	6.8	5.3	5.3	6.9	5.0	8.4	5.4	6.6
96/97	38.7	4.1	4.2	5.3	5.4	2.5	3.1	3.8	4.6
97/98	26.5	4.1	5.3	3.0	5.4	3.7	7.6	3.5	5.7
98/99	22.9	2.7	8.5	4.5	5.9	1.9	3.8	3.0	6.5

From these numbers it appears that those for the G-storms show no correspondence with the sunspot-numbers, also that those for the S-storms show a correspondence which is emphasised according as the intensity increases, and finally that the S-storms show a maximum when the G-storms have a minimum and the reverse.

This latter fact is apparently caused by the circumstance of the storms hiding each other, the G-storms being eclipsed by the S-storms in a higher degree during greater activity of the sun, than the S-storms by the G-storms. Indeed a simple inspection of the diagrams

shows that the agitation of G-storms is greater during a sunspot maximum, than in minimum-years. Also in maximum-years the S-storms of intensity 1, are hidden by their stronger brothers to such an extent, that the eleven-yearly periodicity is nearly the reverse for them.

Annual distribution of S- and G-storms.
(Only the uninterrupted period April 1, 1883—April 1, 1899 has been considered).

Month	Numbers	
	S.	G.
January	31	54
February	31	53
March	29	60
April	24	57
May	24	61
June	27	51
July	31	61
August	29	47
September	32	55
October	31	64
November	22	58
December	18	58

A strong difference in behaviour between G- and S-storms can be noticed. The G-storms have no annual periodicity as to their frequency, whereas the S-storms show a strong one.

This points, just like the daily periodicity of commencement, either to a different origin, or to a changing tendency of the development of the S-impulse during the day and year.

Comparison with Greenwich-storms.

MAUNDER derives from the reproduction of storms published in the volumes of the Greenwich Observations a maximum at 6^h p. m. and

from the original recording-sheets on the contrary at 1^h p. m. The cause of this discrepancy he finds in the manner he looked for the commencement. He writes: "the times when the phases of diurnal disturbance are most strongly marked are naturally most often taken as the times of commencement."

At Greenwich these phases of agitation are most prominent at 1^h p. m. and 6^h p. m.

As I assumed for the hour of beginning the first increase of agitation it is clear my times of beginning are on an average much earlier.

Thus the difference shown by the hourly distribution of commencement between the Greenwich- and the Batavia-list, may be ascribed chiefly to difference of interpretation.

As appears from the figures given above, compared with those for Greenwich the annual periodicity is quite the same for both the northerly and the equatorial stations, which differ no less than 60 degrees in latitude. But the Greenwich dates, quoted from a complete magnetic calendar, prepared by Mr. ELLIS and extending from 1848 to 1902 give no separation of G- and S-storms. Thus it is not possible to decide whether at Greenwich the G-storms lack an annual periodicity in their frequency.

The impulse at the start.

The material at present at my disposal for investigating the features of this phenomenon in other places on earth, is very small.

Notwithstanding this I may conclude: *that this phenomenon is of great constancy in features all over the earth, and consequently a phenomenon of great interest, which might teach us much about the manner the S-storms reach the earth.*

Description of its features for some places.

Greenwich. According to the reduced reproductions of disturbance-curves published in the volumes of the Greenwich-Observations, the impulse consists of a sudden movement in H, D and Z, instantly followed by the reversed movement, the latter being considerably greater. The direction of the movement is always the same.

I have measured 34 cases and have found on an average:

H Δ D	Δ H	Δ Z
25 γ W	+ 77 γ	+ 39 γ (1 γ = 0.00001 C.G.S.).

Batavia. The preceding impulse is missing for H and Z, only for D it is often present.

Here also the direction of the movement is constant.

35 cases for the years 1891 and '92 gave on an average for the

magnitude of digression	$H\Delta D = 9\gamma W$;	$\Delta H = +45\gamma$;	$\Delta Z = -16\gamma$.
duration „ „	3.5 min.;	5 min.;	12 min.

Though the movement of H and D are not sudden in absolute sense, that for Z is too gradual to justify the application of the word sudden to it.

The reproductions of disturbance-curves for Potsdam and Zi Ka Wei also show some cases of the preceding impulse. At both stations the direction of the movement is remarkably constant. In the publications of the Cape Hoorn observations (1882/83) I found three cases exposing also a constant direction.

Summarizing them, we have:

Station	H	D	Z
Potsdam	+	W	—
Greenwich	+	W	+
Zi Ka Wei	+	E	—
Batavia	+	W	—
Cape Hoorn	+	W	—

Consequently with one exception for D and one for Z we find that: *the commencing impulse of the S-storms is the reverse of the vector of postturbation, being deflected however to the West of it.*

Suppositions concerning the origin of disturbance.

The hypothesis on the existence of defined conical streams of electric energy, which strike the earth, though not quite new, has obtained increased plausibility by MAUNDER's results. From the statistics based on the record of disturbances at Batavia it might be concluded that it is chiefly the S-storms that find their origin in the sudden encounter of the earth with such a stream.

And as the earth is first struck at its sunset-arc, it is not impossible that the G-storms, which begin by preference shortly after sunrise and have no annual periodicity in their frequency as the S-storms have, are only partly caused by these encounters.

When in the case of the streams we admit that energy progresses from the sun in the form of negative electrons, we might think the G-storms find their origin by electrified particles being propagated by the light-pressure according to the theory set forth by Sv. ARRHENIUS.

Further we may suppose, that when the earth has received a charge the following development of the storm is the same as it is

dependent on the local hour only. ARRHENIUS has already given an explanation of the nocturnal maximum.

In recent times it has often been attempted to explain magnetic fluctuations by the movement of electric charge through the higher layers of the atmosphere. (SCHUSTER, VAN BEZOLD, SCHMIDT, BIGELOW).

The remarkable analogies which are met with in many cases between the streamfield of atmospherical circulations and the fields of magnetic fluctuations, lead to such speculations.

I believe it is allowed to hazard analogous speculations concerning the cause of the beginning of impulse and postturbation.

We may suppose the streams to contain negative electrons. When they strike the earth the outer layers will be charged with negative electricity. These outer layers do not rotate in 24 hours, but in a longer time increasing with their height.

So a countercurrent of E—W direction charged with negative electrons will originate, causing an increase of H and a decrease of Z. The electrons, however, on entering the magnetic field of the earth, will follow the lines of force towards the magnetic south pole (the positive pole). The movement of negative electrons along the lines of force has been fairly well proved, as is well known, by the aurorarays.

By this movement, the current of electricity will become NE—SW and a westerly deflecting S-impulse will be the consequence.

The sudden charge of the extreme layers of the atmosphere with negative electricity, will attract the positive ions, with which the high layers may be supposed to be charged, to still higher layers.

These positive ions will thus enter into a faster moving counter current, and a positive charged counter current will be the consequence.

These ions will move along the lines of force towards the north, but much slower than the negative electrons, and therefore the resulting deflection of the magnetic force caused by such a + current viz. a force contrary to the ordinary one, will be of no appreciable magnitude. It is conceivable that the effect, which accordingly is in the same sense as the postturbation, will develop in a more gradual manner than the commencing impulse of the S-storms; moreover we may understand that it disappears still more gradually in proportion as the negative electrons again leave the earth or are neutralised by positive ions.

Only we should expect the current to follow the latitude-parallels and accordingly the vectors of postturbation to point to the true south and not to the southerly end of the earth's mean magnetic axis.

Perhaps we may find an explanation for this fact in the influence no doubt exerted by the earth's mean magnetic field and the distribution of positive ions in the atmosphere.

These speculations are indeed very rough, but they have one great advantage, viz. to avoid the difficulty, raised by Lord KELVIN, of allowing an expenditure of the sun's energy causing magnetic disturbances, much too great to be admitted.

CHREE (Terr. Magnet. X, p. 9) points to the fact, that also MAUNDER's defined streams require far too great an expenditure of energy.

According to my opinion we have only to deal with the charge received at the moment of the impulse, and by accepting an intermittent emission of the sun's energy, it is not necessary to integrate it over the entire time between one or more returns of the stream.

Part of the energy is also supplied by the rotation-energy of the earth; and it is curious to remark, that by such an influence the rotation of the earth would be lengthened for a minute fraction during a magnetic storm.

Chemistry. — "*Nitration of meta-substituted phenols*". By Dr. J. J. BLANKSMA. (Communicated by Prof. HOLLEMAN).

(Communicated in the meeting of September 30, 1906).

Some years ago [1]) I pointed out that by nitration of meta-nitrophenol and of 3-5-dinitrophenol tetra- and pentanitrophenol are formed. This showed that the NO_2-groups in the m-position do not prevent the further substitution of the H-atoms in the o- and p-position by other groups. I have now endeavoured to increase these two cases by a few more and have therefore examined the behaviour of some m-substituted phenols which contain, besides a NO_2-group in the m-position, a second group in the m-position, namely of

$$C_6H_3 . OH . NO_2 . (CH_3, OH, OCH_3, OC_2H_5, Cl, Br)\ 1 . 3 . 5.$$

Of these phenols the 5-nitro-m-cresol [2]) and the monomethylether of 5-nitroresorcinol [3]) were known. The still unknown phenols were made as follows:

The 5-nitroresorcinol (m.p. 158°) from its above cited monomethylether by heating for five hours at 160° with (30%) HCl, or by reduction of 3-5-dinitrophenol with ammonium sulphide to 5-nitro-

[1]) These Proc. Febr. 22, 1902. Rec. **21**. 241.

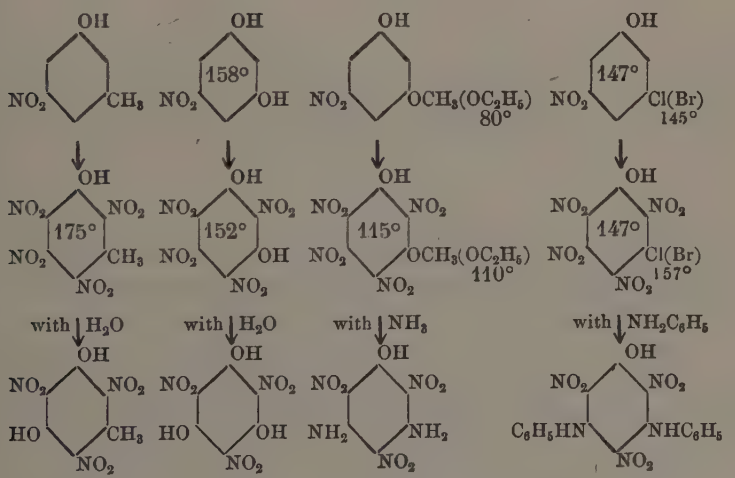
[2]) NEVILE en WINTHER Ber. **15**. 2986.

[3]) H. VERMEULEN Rec. **25**. 26.

3-aminophenol (m.p. 165°) and substitution of the NH_2-group in this substance by OH.

The monoethylether of 5-nitroresorcinol (m.p. 80°) was prepared (quite analogous to the methylether) from 5-nitro-3-aminophenetol; the 3-Cl (Br) 5-nitrophenol was obtained by substituting the NH_2-group in the 5-nitro-3-aminoanisol by Cl (Br) according to SANDMEYER and then heating the 3-Cl (Br)-5-nitroanisol so obtained m.p. (101° [1]) and 88°); with HCl as directed. We then obtain, in addition to CH_3Cl, the desired product 3-Cl (Br) 5-nitrophenol (m.p. 147° and 145°).

The 3-5-substituted phenols so obtained readily assimilate three atoms of bromine on treatment with bromine water and three nitro-groups are introduced on nitration with HNO_3 (Sp. gr. 1.52) and H_2SO_4. These last compounds, which all contain four NO_2-groups resemble picric acid, tetra- and pentanitrophenol. From a mixture of nitric and sulphuric acids they crystallise as colourless crystals which are turned yellow by a small quantity of water; the compounds have a bitter taste, an acid reaction and communicate a strong yellow colour to organic tissues (the skin), others strongly attach the skin and all are possessed of explosive properties owing to the presence of four NO_2-groups [2]).



In this scheme are given only the melting points of the as yet unknown compounds.

Tetranitro-m. cresol yields on boiling with water trinitroorcinol;

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[1]) 91° according to DE KOCK Rec. 20, 113.

[2]) A comparative research as to these properties in the different compounds has not yet been instituted.

in the same manner, tetranitroresorcinol [1]) yields trinitrophloroglucinol; tetranitrochloro- and bromophenol also yield trinitrophloroglucinol on boiling with water or, more readily, with Na_2CO_3 solution. By the action of NH_3 or $NH_2C_6H_5$ etc. in alcoholic solution various other products are obtained, such as those substances included in the scheme which have been obtained previously from pentanitrophenol [2]). We also see that water or alcohol cannot serve as a solvent for the purpose of recrystallising these compounds but that chloroform or carbon tetrachloride may be used.

If, in the above cited 3-5-substituted phenols the OH-group is substituted by OCH_3 it is not possible to introduce three nitro-groups. For instance the dimethylether of 5-nitroresorcinol yields two isomeric trinitroresorcinoldimethylethers (principally those with the melting point 195°, just as in the nitration of 5-nitro-m-xylene) [3]); similarly, the methylether of 5-nitro-m-cresol (m.p. 70°) yields the methylethers of three isomeric trinitro-m-cresols, principally the compound with m.p. 139°. The constitution of these substances is not yet determined.

Amsterdam, September 1906.

Chemistry. — Prof. HOLLEMAN presents a communication from himself and Dr. H. A. SIRKS: "*The six isomeric dinitrobenzoic acids.*"

(Communicated in the meeting of September 29, 1906).

Complete sets of isomeric benzene derivatives $C_6H_3A_2B$ have been studied but little up to the present; yet, for a closer understanding of those derivatives, it must be deemed of great importance to subject the six possible isomers of which such sets consist, to a comparative investigation. A contribution hereto is the investigation of the six isomeric dinitrobenzoic acids which Dr. SIRKS has executed under my directions.

The considerations which guided me in the choice of this series

[1]) According to HENRIQUES (Ann. Chem. **215**, 335), tetranitroresorcinol (m.p. 166°) is formed by the nitration of 2-5-dinitrophenol. In BEILSTEIN's manual (vol. II, 926) a reasonable doubt is thrown on the correctness of this observation. The substance obtained has probably been an impure trinitroresorcinol formed by the action of water on the primary formed tetranitrophenol. (Rec. **21**, 258).

[2]) Rec. **21**, 264.

[3]) Rec. **25**, 165.

of isomers were the following. Firstly, all six isomers were known, although the mode of preparation of some of them left much to be desired. Secondly, this series gave an opportunity to test V. MEYER's "ester rule" with a much more extensive material than hitherto and to study what influence is exercised by the presence of two groups present in the different positions in the core, on the esterification velocity, and to compare this with that velocity in the monosubstituted benzoic acids. Thirdly, the dissociation constants of these acids could be subjected to a comparative research and their values connected with those of the esterification constants. Finally, the melting points and sp. gr. of the acids and their esters could be investigated in their relation to these same constants in other such series.

The six dinitrobenzoic acids were prepared as follows. The symmetric acid 1, 3, 5, (1 always indicates the position of the carboxyl group) was obtained by nitration of benzoic acid or of *m*-nitrobenzoic acid. All the others were prepared by oxidation of the corresponding dinitrotoluenes. This oxidation was carried out partly by permanganate in sulphuric acid solution, partly by prolonged boiling with nitric acid (sp. gr. 1.4) in a reflux apparatus.

We had to prepare ourselves three of the dinitrotoluenes, namely, (1,3,4), (1,3,6) and (1,3,2), (CH_3 on 1); (1.2,4) and (1,2,6) are commercial articles whilst (1,3,5) was not wanted because the orresponding acid, as already stated, was readily accessible by direct nitration of benzoic acid. As will be seen the three dinitrotoluenes which had to be prepared are all derivatives of *m*-nitrotoluene and it was, therefore, tried which of those might be obtained by a further nitration of the same.

m-Nitrotoluene, which may now be obtained from DE HAEN in a pure condition and at a reasonable price was, therefore, treated with a mixture of nitric and sulphuric acids at 50°. On cooling the nitration-product a considerable amount of 1,3,4 dinitrotoluene crystallised out, which could be still further increased by fractionated distillation in vacuo of the liquid portion; the highest fractions always became solid and again yielded this dinitrotoluene, so that finally about 65 grms. of dinitrotoluene (1,3,4) were obtained from 100 grms. of *m*-nitrotoluene,

As the fractions with a lower boiling point, although almost free from dinitrotoluene (1,3,4), did not solidify on cooling, it was thought probable that they might contain, besides a little of the above dinitrotoluene, more than one of the other isomers, whose formation in the nitration of *m*-nitrotoluene is theoretically possible. If we consider

that, in the many cases which I have investigated, the presence of 1% of an isomer causes about 0.5° depression in the melting point, the fact that the oil did not solidify till considerably below 0° and again melted at a slight elevation of temperature whilst the pure isomers did not liquefy till 60° or above, cannot be explained by the presence of relatively small quantities of 1,3,4-dinitrotoluene in presence of *one* other constituent, but it must be supposed to consist of a ternary system. This was verified when the fractionation was continued still further; soon, the fractions with the lowest boiling points began to solidify on cooling, or slowly even at the ordinary temperature, and the solidified substance proved to be 1,2,3 — dinitrotoluene. The fractionation combined with the freezing of the different fractions then caused the isolation of a third isomer namely 1,3,6 — dinitrotoluene, so that the three isomeric dinitrotoluenes desired had thus all been obtained by the nitration of *m*-nitrotoluene. The fourth possible isomer (1,3,5) could not be observed even after continued fractionation and freezing.

As regards the relative quantities in which the three isomers, detected in the nitration product, are formed, it may be mentioned that this product consists of more than one half of 1, 3, 4-dinitrotoluene, whilst (1, 2, 3) seems to occur in larger quantities than (1, 3, 6), as the isolation of the latter in sufficient quantity gave the most trouble.

The *corrected solidifying points* of the dinitrotoluenes (the sixth, symmetric one was prepared by BEILSTEIN's method A. **158**, 341 in order to complete the series) were determined as follows. Those of the dinitrobenzoic acids and of their ethyl esters are also included in the subjoined table.

	3.4	3.5	2.3	2.5	2.6	2.4
Dinitrotoluenes	58.3	92.6	59.3	50.2	65.2	70.1
Dinitrobenzoic acids	163.3	206.8	204.1	179.0	206.4	180.9
Ethyl esters	71.0	92.9	88.4	68.8	74.7	40.2

The *specific gravities* of the dinitrotoluenes and the ethyl esters were determined by means of EYKMAN's picnometer at 111°.0 with the following result:

CO_2H or CH_3 on 1	toluenes	esters
3.4	1.2594	1.2791
3.5	1.2772	1.2935
2.3	1.2625	1.2825
2.5	1.2820	1.2859
2.4	1.2860	1.2858
2.6	1.2823	1.2923

Water at 4° as unity. Corrected for upward atmospheric pressure and for expansion of glass.

Conductivity power. This was determined in the usual manner with a Wheatstone-bridge and telephone at 25° and at 40°. As the acids are soluble in water with difficulty $v = 100$ or 200 was taken as initial concentration; the end concentration was $v = 800$ or 1600. In the subjoined table the dissociation constants are shown.

Dinitrobenzoic acids		3.4	3.5	2.3	2.5	2.4	2.6
$K = 100\,k$	at 25°	0.163	0.163	1.44	2.64	3.85	8.15
	at 40°	0.171	0.177	1.38	2.16	3.20	7.57

On comparing these figures it is at once evident that the acids with ortho-placed nitro-group possess a much greater dissociation constant than the other two, so that in this respect, they may be divided into two groups. In the acids without an ortho-placed nitro-group, the value of the dissociation constant is fairly well the same. In the other four, the position of the second group seems to cause fairly large differences. That second group increases the said constant most when it is also placed ortho: in $N/_{100}$ solution 2-6-dinitro-benzoic acid is ionised already to the extent of 90 %. Again, a NO_2-group in the para-position increases the dissociation constant more than one in the meta-position; and for the two acids 2,3 and 2,5 which both have the second group in the meta-position, K is considerably larger for 2,5, therefore for the non-vicinal acid than for the vicinal one, so that here an influence is exercised, not only by the position of the

groups in itself, but also by their position in regard to each other. It also follows that OSTWALD's method for the calculation of the dissociation constants of disubstituted acids from those of the monosubstituted acids cannot be correct as is apparent from the subjoined table:

Dinitrobenzoic acid CO_2H on 1	K calculated	K found
3.4	0.23	0.16
3.5	0.20	0.16
2.4	4.1	3.8
2.5	3.6	2.6
2.3	3.6	1.4
2.6	64	8.1

In the two vicinal acids 2,3 and 2,6 the deviations from the calculated value are particularly large, as I have previously shown for other vicinal substituted acids (Rec. **20**, 363).

In view of the comparison of the figures for the dissociation constants of these acids and for their esterification constants, it seemed desirable to have also an opinion as to the molecular conductivity of these acids in alcoholic solution. They were, therefore dissolved in 95 vol. % alcohol to a $N./_{200}$ solution and the conductivity power of those liquids was determined at 25°. The subjoined table shows the values found and also those of the aqueous solutions of the same concentration and temperature:

dinitrobenzoic acids	3.4	3.5	2.3	2.5	2.6	2.4
μ_{200} in alcoh. sol.	1.1	1.15	1.75	2.25	2.7	2.9
μ_{200} in aqueous sol.	161.5	162.5	293	321	355.5	335.5

from which it appears that also in alcoholic solution the acids with an ortho-placed nitro-group are more ionised than the others.

Esterification velocity. The method followed was that of GOLDSCHMIDT, who dissolved the acid in a large excess of alcohol and used hydrochloric acid as catalyzer. The alcoholic hydrochloric acid used here

was 0,455 normal. Kept at the ordinary temperature it did not change its titre perceptibly for many months. As GOLDSCHMIDT showed that the constants are proportionate to the concentration of the catalyzer, they were all recalculated to a concentration of normal hydrochloric acid. Owing to the large excess of alcohol the equation for unimolecular reaction could be applied. The velocity measurements were executed at 25°, 40° and 50°. At these last two temperatures, the titre of the alcoholic acid very slowly receded (formation of ethylchloride) and a correction had, therefore, to be applied. The strength of the alcohol used was 98.2 % by volume.

In order to be able to compare not only the esterification-constants E of the dinitrobenzoic acids with each other but also with those of benzoic acid and its mononitroderivatives, the constants for those acids were determined at 25° under exactly the same circumstances as in the case of the dinitroacids. The results obtained are shown in the subjoined table:

Acids	E at 25°	E at 40°	E at 50°
benzoic acid	0.0132	—	—
m. NO_2 »	0.0071	—	—
o. » »	0.0010	—	—
3.4 dinitro »	0.0086	0.033	0.077
3.5 » »	0.0053	0.028	0.060
2.3 » »	0.0005	0.0025	0.0071
2.5 » »	0.0003	0.0027	0.0076
2.4 » »	0.0002	0.0017	0.0056
2.6 » »	unmeasurably small		

As will be seen, E is by far the largest for benzoic acid and each subsequent substitution decreases its value.

On perusing this table it is at once evident that in the dinitrobenzoic acids two groups can be distinguished. Those with an ortho-placed nitro-group have a much *smaller* constant than the other two. *Whilst therefore the dissociation constant for acids with an ortho-placed nitro-group is the largest their esterification constant is the smallest.* As shown from the subjoined table, this phenomenon proceeds quite parallel; the acids whose dissociation constant is greatest have the smallest esterification constant and vice versa.

Dinitrobenzoic acids	diss. const. at 40°	esterif. const at 40°
3.4	0.171	0.033
3.5	0.177	0.028
2.3	1.38	0.0025
2.5	2.16	0.0027
2.4	3.20	0.0017
2.6	7.6	< 0.0001

On perusing the literature we have found that this regularity does not exist in this series of dinitrobenzoic acids only, but is observed in a comparatively large number of cases. The strongest acids are the most slowly esterified. This might lead us to the conclusion that in the esterification by alcoholic hydrochloric acid it is not the ionised but the unsplit molecules of the acids which take part in the reaction.

A more detailed account of this investigation will appear in the Recueil.

Amsterdam
Groningen } Sept. 1906, Laboratory of the University.

Chemistry. — Prof. HOLLEMAN presents a communication from himself and Dr. J. HUISINGA. *"On the nitration of phthalic acid and isophthalic acid"*.

(Communicated in the meeting of September 29, 1906).

Of phthalic acid, two isomeric monoderivatives are possible, both of which are known particularly by a research of MILLER (A. **208**, 233). Isophthalic acid can yield three isomeric mononitro-acids. Of these, the symmetric acid, which is yielded in the largest quantity during the nitration, is well known. As to the mononitrated by-products formed, the literature contains a difference of opinion ; in any case, there is only made mention of one second mononitro-acid whose structure has remained doubtful.

The investigation of the nitration of phthalic and isophthalic acid was taken up by us in order to determine the relative amount of the isomers simultaneously formed, as in the case of the mononitro-

phthalic acids only a rough approximation (by MILLER) was known, whilst in the case of the mononitroisophthalic acids it had yet to be ascertained which isomers are formed there from.

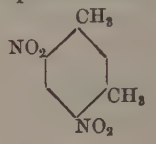
We commenced by preparing the five mononitro-acids derived from phtalic acid and isophthalic acid in a perfectly pure condition. In the case of the α- and β-nitrophthalic acids no difficulties were encountered, as the directions of MILLER, save a few unimportant modifications, could be entirely followed. The acids were therefore obtained by nitration of phthalic acid and separation of the isomers.

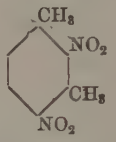
The *symmetric nitro-isophthalic acid* was prepared by nitration of isophthalic acid. It crystallises with 1 mol. of H_2O and melts at 255—256° whilst it is stated in the literature that it crystallises with $1^1/_2$ mol. of H_2O and melts at 248°. At first we hoped that the other two nitroisophthalic acids might be obtained from the motherliquors of this acid. It was, therefore, necessary to obtain the isophthalic acid in a perfectly pure condition, as otherwise it would be doubtful whether the byproducts formed were really derived from isophthalic acid. By oxidation of pure *m*-xylene (from KAHLBAUM) an isophthalic acid was obtained which still contained terephthalic acid which could be removed by preparing the barium salts.

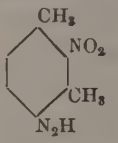
The motherliquors of the symmetric nitro-isophthalic acid appeared, however, to contain such a small quantity of the byproducts that the preparation of the nitro-acids (1, 3, 2) and (1, 3, 4) was out of the question. These were therefore, prepared as follows:

Preparation of asymmetric nitro-isophthalic acid (1, 3, 4). On cautious nitration of *m*-xylene at 0° with nitric acid of sp. gr. 1.48 a mixture is formed of mono- and dinitroxylene which still contains unchanged *m*-xylene. This, on distillation with water vapour, passes over first and when drops of the distillate begin to sink to the bottom of the receiver the latter is changed and the distillation is continued until crystals of dinitroxylene become visible in the condenser. 100 gr. of xylene gave about 85 gr. of mononitroxylol (1, 3, 4).

After rectification of this mononitroxylene (b. p. 238°) it was oxidised in alkaline solution with a slight excess of permanganate; 20 gr. yielded 12 à 13 gr. of acid which, however, consisted of a mixture of nitrotoluylic acid and nitro-isophthalic which could be separated by crystallisation from water. In this way, the as. nitro-isophthalic acid was obtained with a melting point of 245°. In water it is much more soluble than the symmetric acid, namely to the extent of about 1 °/₀. at 25°. Unlike the symmetric acid, it crystallises without water of crystallisation in small, fairly thick, plate-like crystals. It is very readily soluble in hot water, alcohol and ether,

Preparation of the vicinal nitro-isophthalic acid (1, 3, 2). GREVINGK has observed that in the nitration of m-xylene with nitric and sulphuric acid there is formed, besides the symmetric dinitro-m-xylene 

as main product, also the vicinal isomer 

. On reduction with hydrogen sulphide both dinitroxylenes pass into nitro-xylidenes which are comparatively easy to separate. The nitro-xylidene 

yields by elimination of the NH_2-group vicinal nitro-m-xylene. Whilst however, GREVINGK states that he obtained a yield of 25% of vicinal nitroxylidene we have never obtained more than a few per cent of the same so that the preparation of vicinal nitro-m-xylene in this manner is a very tedious one, at least when large quantities are required. When it appeared that the "fabrique de produits chimiques de Thann et Mulhouse" exported this nitroxylene, the oxidation, although to some extent with material of our own manufacture, has been mainly carried out with the commercial product. This oxidation was also done with permanganate in alkaline solution. The vicinal nitro-isophthalic acid is a compound soluble with great difficulty in cold, but fairly soluble in hot water, crystallising in small beautiful, shining needles, which melt at 300°. It crystallises without any water of crystallisation and is readily soluble in alcohol and ether, from which it is again deposited in small needles.

The three possible mononitroisophthalic acids having now been obtained, we could take in hand the problem to ascertain the nature of the byproduct formed in the nitration of isopthalic acid. After the bulk of the nitroisophtalic acid formed had been removed by crystallisation, a residue was left which was far more soluble in water than this acid, which pointed to the presence of the asymmetric nitro-acid and which, indeed, could be separated by fractional crystallisation. We will see presently how it was ascertained that the nitration product was really only a mixture of the symmetric and the asymmetric acid.

As in the determination of the relative quantities in which the nitration products are formed, use was made of solubility determina

tions, we first give the solubilities in water at 25° of the five nitrophthalic acids, in parts per 100.

α-nitrophtalic acid	β-nitrophthalic acid
2.048	very soluble
symmetric nitroisophthalic acid with water of crystallisation.	Asymmetric nitroisophthalic acid
0.157	0.967

Vicinal nitroisophthalic acid
0.216

Quantitative nitration of phthalic acid. This was done with absolute nitric acid. It appeared that it proceeded very slowly even at 30°, and therefore the phthalic acid was left in contact with six times the quantity of nitric acid for three weeks. After dilution with water the acid was expelled by heating on a waterbath or else evaporated over burnt lime. The solid residue was then reduced to a fine powder and freed from the last traces of nitric acid by prolonged heating at 110°. As under the said circumstances the mononitrophthalic acids are not nitrated any further, it could be ascertained by titration whether all the phthalic acid had been converted into the mononitro-acid; the product had but a very slight yellow colour so that a contamination could be quite neglected. Of the product, now ready for analysis, different quantities were weighed and each time introduced into 100 c.c. of water, and after adding an excess of α-nitrophthalic acid they were placed in the shaking apparatus. The amount of acid dissolved was determined by titration and from these figures the content in β-acid was calculated by making use of a table which had been constructed previously and in which was indicated which β-nitroacid contents correspond with a definite titre of a solution so obtained. As the mean of four very concordant observations it was found that in the nitration of phthalic acid with absolute nitric acid at 30° is formed:

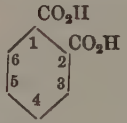
49.5 % α- and 50.5 % β-nitrophthalic acid.

The quantitatwe nitration of isophthalic acid was done in the same manner as that of phthalic acid; here also, a few weeks were required for the complete nitration at 30°. The contamination with yellow impurities could again be quite neglected as a but very faintly coloured nitration product was obtained. This nitration product so obtained contains the anhydrous symmetrical nitroisophthalic acid, so that in the solubility determinations by which its composition was determined the hydrated acid had to be employed as the anhydrous acid takes up water but very slowly and has a greater solubility. The

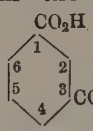
determination of the total amount of byproduct showed that this had formed to the extent of 3.1 % only. The qualitative investigation had shown already that this contains the asymmetric acid, and that it consists of this solely was proved in the following manner. If the 3.1 % found were indeed simply asymmetric acid, a solution, obtained by shaking 100 grams of water with excess of symmetric and vicinal acid + 1 gram of nitration product (containing 0.031 gram of asymmetric acid), ought to have the same titre as a solution obtained by shaking 100 grams of water with excess of both acids + 0.031 gram of asymmetric acid. If on the other hand the nitration product also contained vicinal acid, therefore less than 0.031 gram of asymmetric acid, the titre ought to have been found less. This however, was not the case, which shows that the asymmetric acid is the sole byproduct. The result, therefore, is that in the nitration of isophthalic acid with absolute nitric acid at 30° there is formed:

96.9 % of symmetric and 3.1 % of asymmetric nitroisophtalic acid.

If we compare the above results with that of the nitration of benzoic acid where (at 30°) is formed 22.3 % ortho-, 76,5 % meta- and 1.2% paranitrobenzoic acid the following is noticed.

As in phthalic acid  the positions 3 and 6 are meta in

regard to the one carboxyl and ortho in regard to the other and the positions 4 and 5 are also meta in regard to the one carboxyl but para in regard to the other it might be expected from my theories that the α-acid (the vicinal) is the main product and the β-acid the byproduct, because in the latter the nitro-group must be directed by one of the carboxyles towards para and because p-nitrobenzoic acid is formed only in very small quantity in the nitration of benzoic acid. As regards the isophthalic acid



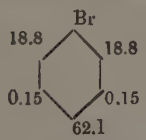
it might be expected that the chief product will be symmetric acid but that there will also be byproducts (1, 3, 2) and (1, 3, 4) the first in the largest quantity, although it should be remembered that a nitro-group seems to meet with great resistance if it must take a position between two other groups.

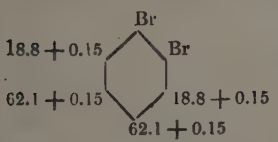
As regards the nitration of isophthalic acid the result of the above investigation is fairly satisfactory, although the total absence of the vicinal nitroisophthalie acid is somewhat remarkable. In the case of phthalic acid this is true in a less degree as about equal quantities are formed of the two possible isomers.

In his dissertation, Dr. HUISINGA has now endeavoured to calculate,

more accurately than before, from the relative proportion in which the isomers C_6H_4AC and C_6H_4BC are formed by the introduction of C in C_6H_5A or C_6H_5B, in what proportion the isomers C_6H_3ABC are formed by the introduction of C in C_6H_4AB. He observes first of all that in a substance C_6H_5A there are two ortho and two meta positions against one para position so that if the relation of the isomers is as C_6H_5AC p : q : r (ortho, meta, para) this relation for each of the ortho and meta positions and for the para position will be $^1/_2$ p : $^1/_2$ q : r

He further gives the preference to an addition of these figures of proportion instead of a multiplication, which had been used by me up to the present in the prediction of these isomers. He prefers the addition because he considers the figures of proportion to be proportional to the directing forces which are exercised by the groups A and B on the other positions of the core and that the cooperation of such directing forces on one H-atom should be represented by a sum. But only the proportion of those directing forces are known and not their absolute value; the force which, in the nitration of nitrobenzene, pushes the NO_2-group towards the *m*-position may be of quite a different order than the force which in the nitration of benzoic acid directs the same group towards the m-position. Therefore the figures which represent the directing forces (or are proportionate to the same) of two different groups cannot always be simply added together; this then will be permissible only when the two substituents present are equal.

As an example of his method of calculation the following may be mentioned. As in the nitration of bromobenzene 37.6% ortho, 62.1% para and 0.3% of meta nitrobromobenzene is formed, the substitution in the different positions of the benzene core takes place in the proportion 

; for the proportion in which the isomers are formed in the nitration of o-dibromobenzene the calculation gives 

or 62.25 : 18.95; or 76.7 % asymmetric nitro-o-dibromobenzene and 23.3 % vicinal whilst the experiment gave 81.3 % asymmetric and 18.3 % vicinal.

It cannot be denied that in a number of cases this method of calculation gives figures which approach to the experimental ones a

good deal more than those obtained formerly when the undivided figures were simply multiplied. But on the other hand there are other cases, particularly those in which a metasubstituted substance is nitrated, where this calculation does not agree with the experiment by a long way. If we take into account the figures of proportion for the single positions we obtain as a rule a much better approach to the figures observed by means of the products than with the sums, even in the case where the two substituents present are unequal, when HUISINGA's method of calculation cannot be applied. The proof thereof is laid down in the subjoined table which gives the figures of proportion in which the isomeric nitroderiva tives are formed from the substances at the top of the columns, with the figures obtained from both the sums and the products.

	Cl:Cl ortho	Cl:Cl meta	Br:Br ortho	Br:Br meta	$Co_2H:CO_2H$ ortho	$CO_2H:CO_2H$ meta
found	7:93	4:96	18.3:81.7	4.6:95.4	49.5:50.5	3.1:96.9
product	18:82	9:91	23.3:76.7	13:87	82 :18	*10.6:89 4
sum.	18:82	15:85	23.3:76.7	19:81	55.6:44.4	*38 :62

*totalquantity byproduct.

	CO_2H : Cl ortho	CO_2H : Cl meta	CO_2H : Br ortho	CO_2H : Br meta
found	16.0:84.0	8.7 : 91.3	19.7:80.3	11.4:88.6
product	17.7:82.3	17.7:82.3	23.3:76.7	23.3:76.7

A fuller account of this investigation will appear in the Recueil. *Amsterdam*, org. lab. Univ. 1906.

Astronomy. — "*The relation between the spectra and the colours of the stars.*" By Dr. A. PANNEKOEK. (Communicated by Prof. H. G. VAN DE SANDE BAKHUYZEN).

(Communicated in the meeting of September 29, 1906).

The close relation between a star's colour and its spectrum has long been known. The stars of the 1st, 2d and 3d types are usually called the white, the yellow and the red stars, although accurately spoken the colour of the so-called yellow stars is a very whitish unsaturated yellow colour and that of the so-called red stars is deep yellow mixed with very little red. In a paper read at Dusseldorf [1]) in

[1]) Die Farben der Gestirne. Mittheilungen der V. A. P. Jahrg. 10. S. 117.

1900 we showed that in the different glowing conditions the colours must succeed each other in this order. If for a given high temperature we accept the colour to be white, we find that with decrease of temperature the colours in the triangular diagram of colours make a curve which from white first goes directly to yellow of λ 587 but which, as the colour becomes deeper, bends towards the red and corresponds to light of greater wavelength. With increase of temperature, on the contrary, the line of colour runs from white to the opposite side, to the blue of λ 466.

Because the colours which are produced by white light after having been subjected to different degrees of atmospheric absorption, also follow about this same line, we may expect that the colours of the self-luminous celestial bodies will in general lie on this line or near it; they are determined on this line by one coordinate, one number. This renders it comprehensible why on the one hand the designation by means of letters and words, or the measurement with ZÖLLNER's colorimeter, which produces quite different colours, has given so few satisfactory results, and on the other hand why the scale of SCHMIDT, who designates the colours by one series of figures, where 0 is white, 4 yellow, and 10 red has proved to be the best to work with. After this method has been drawn up the best and most complete list of stellar colours, published in 1900 by H. OSTHOFF at Cologne, in the A. N. Bd. 153 (Nr. 3657—58). This list in which the colours of all stars to the 5th magnitude are given, down to a tenth class of colour, and which was the fruit of systematic estimates during 14 years, enables us to accurately determine the relation between spectrum and colour.

In a former paper [1]) we remarked that we did not know where in the continuous series of spectra of the Oriontype and the first type we have to look for the highest temperature or at any rate the greatest luminosity. We may assume that it will be there where the colour is whitest; the spectral-photometric measurements, to which we have alluded in that paper, are still wanting, but for this purpose we can also avail ourselves advantageously of estimates of colour; this has been the reason for the investigation of which the results follow here.

In this case where we required a specification of the spectra, as detailed as possible, to serve as an argument for the colour, we have naturally used again MAURY's classes. In order, however, to determine a mean colour for each class we must correct the colours

[1]) The luminosity of stars of different types of spectrum. Proceedings of June 30 1906 p. 134.

observed for two modifying influences, viz. the influence of the brightness and that of the altitude above the horizon. Quantitatively nothing is known about the values of these influences; experiments of OSTHOFF himself to determine the influence of the brightness have as yet yielded few results. Therefore we must derive them here from the material of stellar colours themselves, which serve for our investigation; this may be done in the very probable assumption that the real colour within each spectral class is an almost constant value and is independent of brightness.

§ 2. The stars of OSTHOFF's list which occur in the spectral catalogue of MAURY, were arranged according to their classes and then (excluding those which are marked *c*, *ac*, *C*, *P* or *L*, as was always done in this investigation) always taking together some classes, we classified them according to their brightness and combined their magnitudes and classes to mean values. These mean values must show the influence of the brightness on the colour; they are given in the following tables:

Classe III—VI		Classe VII—VIII		Classe IX—XII	
Mg.	Col.	Mg.	Col.	Mg.	Col.
1.78	1.46 (5)	0.1	1.2 (3)	1.0	2.7 (2)
2.80	2.27 (6)	2.4	1.83 (6)	2.69	2.97 (9)
3.35	1.96 (5)	3.17	2.59 (7)	3.18	3.06 (8)
3.70	2.86 (7)	3.55	2.57 (6)	3.65	3.73 (10)
4.00	2.47 (8)	3.82	2.95 (6)	3.85	3.40 (8)
4.15	2.91 (7)	4.00	2.86 (5)	4.10	3.69 (9)
4.50	2.60 (9)	4.10	2.60 (7)	4.29	4.17 (7)
4.95	2.42 (11)	4.20	2.50 (5)	4.65	3.79 (8)
		4.36	2.96 (5)	5.10	3.34 (9)
		4.62	2.72 (4)		
		4.96	2.66 (5)		

Classe XIII—XIV		Classe XV		Classe XVI—XVIII	
Mg.	Col.	Mg.	Col.	Mg.	Col.
0.2	3.4 (1)	0.7	4.5 (2)	0.95	6.45 (2)
3.07	4.71 (7)	2.12	5.50 (6)	2 50	6.40 (6)
3.54	4.61 (7)	2.92	5.66 (9)	3.22	6.65 (6)
3.98	4.72 (9)	3.37	5.74 (9)	3.72	6.65 (4)
4 24	4.88 (8)	3.55	5.46 (9)	4.15	6.75 (6)
4.84	4.88 (8)	3.75	5.71 (8)	4.63	7.07 (7)
		3.90	5.55 (10)	4.88	7.22 (9)
		4.00	5.70 (7)	5.28	7.22 (8)
		4.14	5.85 (11)		
		4.45	6.08 (6)		
		4.87	6.43 (7)		

In all these series we clearly see an increasing deepening of colour with decreasing brightness. We have tried to represent the colour as a linear function of the magnitude; and by a graphical method we found:

Cl.	III—VI	$c = 2.15 + 0.35$	$(m - 3)$
„	VII—VIII	$2.27 + 0.36$	„
„	IX—XII	$3.17 + 0.39$	„
„	XIII—XIV	$4.45 + 0.42$	„
„	XV	$5.47 + 0.39$	„
„	XVI—XVIII	$6.60 + 0.20$	„

Thus we find about the same coefficient in all groups except in the last. The value of the coefficients is chiefly determined by the difference between the observed colours of the very bright stars of the 1st magnitude and of the greater number of those of the 3d and 4th magnitudes. In order to make the coefficient of the last group agree with the others, it is necessary to assume for the apparent colour of α Tauri and α Orionis 5.6 instead of the real estimates 6,4 and 6,5. It does not do, however, to assume such a large error for these bright and often observed stars; therefore we must for the present accept the discordant coefficient of the red stars as real, although it is difficult at the present to account for it.

If now we combine the results of the five first groups by arranging the deviation of each observed value of c from the constant for the group (the value of c for $m = 3$), according to brightness and deriving thence mean values we find:

m	$c - c_3$	C_1	C_3	$O - C_1$	$O - C_3$
0.3	—1.03	—1.10	—0.91	+.07	—.12
1.6	—0.63	—0.54	—0.47	— 09	— 16
2.91	+0.02	+0.04	0.02	— 02	+ 04
3.73	+0.32	+0.31	+0.27	+ 01	+ 05
4.12	+0.48	+0.40	+0.39	+ 08	+ 09
4.73	+0.50	+0.52	+0.60	— 02	— 10

A linear relation $c = c_3 + 0{,}34\,(m - 3)$ yields the computed values given under C_3 and the differences obs.-comp. $O - C_3$. These are distributed systematically and show the existence of a non-linear relation. A curve, which represents as well as possible the mean values, gives the computed values C_1 and the differences, obs.-comp. $O - C_1$. For a greater brightness the curve gives a greater variation of the colour with the luminosity and for fainter stars a smaller one. In all the six groups, except the fifth and the sixth, we remark that

the last values, which hold for the faintest magnitudes, show a decrease in the colour figures with regard to the preceding ones.

This phenomenon may be accounted for by the existence of the colourless perception of faint sources of light. In faint stars we do not see any colour at all; there the perception of colour disappears almost entirely and there remains only a colourless (i. e. whitish) impression of light. With stars which approach this limit, the impression of colour will be mixed up to a high degree with the colourless impression, and therefore they appear paler and will be indicated by a lower figure. As for the redder stars this colourless impression is relatively much weaker, the paleness of colour for these stars occurs only with a much less degree of brightness; in this manner we explain why the 5th and 6th groups do not show this decrease. Whether in these cases the phenomenon occurs with fainter stars cannot be decided because MAURY's spectral catalogue does not contain fainter stars.

For the practical purpose of reducing the observed colours to one brightness it is about the same which of the two relations is adopted, as long as we keep within certain limits of brightness, for instance between the magnitudes 1 and 5. To facilitate the reduction we have made use of the linear formula given above for the 5 first groups (down to class XV included) while for the redder classes 0,20 has been adopted as the coefficient of brightness.

To explain the long known phenomenon that the colour deepens with decreasing brightness as is shown in the tables on p. HELMHOLTZ in his *Physiologische Optik* has given a theory called "Theorie der kürzesten Linien im Farbensystem". In the diagram of colours in space, where each impression of light is represented by a point of which the 3 coordinates represent the quantities of the elemental colours, red, green, blue, the lines of equal colours are not straight radii through the origin, but curved lines which with increasing distance from the origin bend more and more towards the axes and so diverge more and more from one radius which is straight and represents the "Principalfarbe". Hence in the triangle of colours the points of equal colour diverge the more from the principal colour and run in curved lines towards the sides and the vertices as the triangle of colours is removed farther from the origin, and thus represents a greater brightness. HELMHOLTZ gives as principal colour a certain "yellow-white" to which with extremely great intensity all colours seem to approach. Therefore colours which lie on the blue side of this principal colour must become bluer by fading.

This does not agree with what we have found here, in the supposition that HELMHOLTZ's "yellow-white" is also yellow-white in our scale, i. e. is also represented by a positive number in SCHMIDT's scale. We also find here with the whitest stars that when they become fainter the colour becomes more yellow to just the same degree as with the yellower stars. Now the expression "yellow-white" is vague, but if we consider that what is called white in the scale of SCHMIDT is whiter, that is to say bluer than the light of Sirius, and that the solar light, the standard for white for ordinary optical considerations, if weakened to the brightness of a star, in the scale of SCHMIDT would be called 3 à 4 (Capella 3, 4), then the principal colour, if HELMHOLTZ's theory is true, instead of being yellow-white would still lie on the blue side of the Sirius light.

§ 3. After the colours had thus been reduced to the brightness 3,0, they had still to be freed from the influence of the atmosphere, which makes them redder. This cannot be done with the desired accuracy, because neither time nor altitude are given along with the observations. The influence at high and mean altitudes is probably very small, and the observer is sure to have taken care that most of the stars were observed at a proper altitude (for instance between 30° and 60°). Therefore this correction is only practically important for the few southern stars which always remain near the horizon; in these cases it will be possible to represent the variation of colour by a correction depending on the declination. Instead of the declination of the star we have taken the declination of the B. D.-zone which OSTHOFF has added to his catalogue.

For each spectral class we have determined mean colour-values for all stars north of the equator, and for the stars south of the equator we have formed the deviations from these class-means which then were arranged according to their declination and combined to mean values for groups of stars. We have excluded, however, those classes in which too few northern stars occurred, namely I, II and III.

The means found are:

Zone	Deviation	n.	Curve	Zone	Deviation	n.	Curve
0°0	+0.56	5	+0.05	−9°0	+0.14	5	+0.26
−1 7	+0.35	4	+ 06	−10.2	+0.35	4	+ 32
−3.3	−0.17	6	+ 09	−13.2	+0.33	6	+ 57
−5.0	+0.50	5	+ 12	−15.0	+1.17	6	+ 79
−6.6	+0.22	4	+ 17	−18.2	+0.93	6	+1.32
−8.0	−0.05	5	+ 22				

Through these values we have drawn a curve which from the equator towards the southern declinations ascends steeper and steeper and which gives the values of the last column. According to this curve we have applied the following corrections, for

zone	1°	2°–5°	6°–8°	9°–10°	11°	12°	13°	14°	15°	16°	17°	18° South
neg. corr.	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,1	1,3

We may assume that by these corrections the variation of colour due to atmospheric absorption has at least for the greater part been eliminated.

§ 4. After the two corrections (§ 2 and § 3) had been applied we could determine for all spectral classes the mean values of the colour; they are given in the following table. Class XV was again subdivided into 3 classes according as the spectrum agreed with α Bootis (A) or with α Cassiopeiae (C) or was not accompanied by any such remark; the result shows indeed that here class C is considerably redder than class A while the B's lie between the two.

Class	Colour	Number	Class	Colour	Number
I	2.47	6	XII	3.68	17
II	2.36	10	XIII	4.12	13
III	2.30	9	XIV	4.45	12
IV	1.94	14	*XIV*	5.09	9
IV	1.62	10	XVA	5.18	18
V	2.11	9	XVB	5.35	26
VI	2.16	10	XVC	5 55	31
VII	2.27	23	*XV*	6.34	5
VIII	2.37	34	XVI	6.47	17
IX	2.64	20	XVII	6.80	15
X	3.11	14	XVIII	6.74	15
XI	3.40	9	XIX	6.67	6
XI	3.41	4			

The deviations of the separate values from these mean values give, as a measure for the accuracy of the results, for the mean error of a colour-number, $\sqrt{0,20} = 0,45$; the real accuracy will be greater, however, and the mean error smaller because in these values are also included the errors of the adopted corrections for brightness and declination, the errors which may have been made by MAURY while classifying each star in a definite class, and also the real deviations of the single stars from their class-means.

With 9 stars (out of 355) the deviation exceeds a unit of colour; the reduced colours are here:

β Can. maj.	III 1,2	ε Hydrae	XIII 5,2	η Persei	XV*B* 6,8
o_1 Cygni	IX 1,4	μ Persei	XIV 5,5	11 Urs. min.	XV*B* 6,6
σ Delphini	IX 3,8	o_2 Cygni	*XIV* 6,5	5 Orionis	XVII 7,9

In this investigation we have, as it was said before, excluded the *c*- and *ac*-stars, the *L* (bright lines), the *P* (peculiar spectra) and the *C* (composed spectra). It is important to examine the *c* and the *ac*-stars among them more closely in order to see whether they show a distinct difference in colour from the *a*-stars of the same class-number. In the mean 11 *ac*-stars give a deviation of + 0,1 (from + 0,5 to — 0,3), and 12 *c*-stars + 0,7; so these last ones are a little redder than the *a*-stars. Here, however, the great individual deviations are very striking; the extreme values are:

ϱ Cassiop XIII + 2,5; χ_3 Orionis III + 1,8; 4*H* Camelop *VI* + 2,0; 3*H* Camelop *VI* + 1,5; η Leonis VII — 0,3; β Orionis VI — 1,2,

The differences are very great, but no regularity can be detected.

§ 5. The results found solve a problem which in my former paper remained unsolved, namely where in the continuous series of spectral classes shall we have to look for the maximum of radiating power. The colour-numbers show very distinctly a fall in the first classes, a *minimum* between the 4th and the 5th class and then a continual rise. The stars which in order of evolution directly follow on γ Orionis (μ Aurigae, μ Hydrae, *u* Herculis) have the whitest colours; both the earlier and the later stages of evolution are yellower; classes I and II agree in colour best with class VIII. Therefore, in so far as we are entitled to derive the entire radiation from the colour, the maximum of radiating power lies between the 4th and the 5th class.

The mean colour-numbers for each of the groups formed before are:

Cl. I—III	2.35
IV—V	1.87
VI—VIII	2.30
IX—XII	3.20
XIII—XIV	4.58
XV	5.43
XVI—XIX	6.66

Is it possible to derive from these numbers, even though only approximately, values for the radiating power per unit of surface? The two influences mentioned above which determine the colours of the self-luminous celestial bodies may be subjected to a calculation, if we disregard the specific properties of the composing substances and treat them as abstract theoretical cases. In other words we can investigate the radiation of a perfectly black body and in the absorption neglect the selective absorption in lines and bands in order only to examine the general absorption. As a first approximation this may be deemed sufficient.

In this calculation we have made use of the measurements of A. KÖNIG on the relative quantities of the elemental colours red, green and blue as functions of the wavelength in white sunlight. If for an other source of light we know the relation of the brightness with regard to the former source as a function of the wavelength, we can calculate the quantities of the red, green and blue in this second source of light. If we call the numbers of KÖNIG $R(\lambda)$, $G(\lambda)$, $B(\lambda)$, which are chosen so that

$$\int R(\lambda)\, d\lambda = 1000 \qquad \int G(\lambda)\, d\lambda = 1000 \qquad \int B(\lambda)\, d\lambda = 1000$$

and if $f(\lambda)$ represents the brightness of another source of light, then

$$\int f(\lambda)\, R(\lambda)\, d\lambda \qquad \int f(\lambda)\, G(\lambda)\, d\lambda \qquad \text{and} \qquad \int f(\lambda)\, B(\lambda)\, d\lambda$$

represent the quantities of R, G, and B occurring in this light. As the impression of brightness of a source of light is almost proportional to the quantity of red, this calculation gives at the same time a measure for the optical brightness.

The radiation of a black body may be represented by:

$$\lambda^{-\alpha}\, e^{-\frac{c}{T\lambda}}\, d\lambda$$

where T is the absolute temperature and α and c constants. For two sources of light of different temperatures the relation of the intensities is:

$$f(\lambda) = e^{-\frac{c}{\lambda}\left(\frac{1}{T}-\frac{1}{T_0}\right)} = e^{\frac{b}{\lambda}} = 10^{\frac{b'}{\lambda}},$$

if $b = c\left(\frac{1}{T_0}-\frac{1}{T}\right)$ and $b' = 0.43\, b$. As unit for λ we adopt 0,001 mm; T_0 is supposed to be given, then b' is a function of the variable temperature T only and may be called the degree of glowing with regard to the glowing of a body at a temperature T_0. If we adopt

for b' different values ($c = 15000$ about) [1]), we can calculate for each of them the brightness and colour of the light, as well as the temperature T. We then find for the degrees of glowing $+1$, 0 and -1

$$\begin{array}{rrrrrrr} b' = +1 & 69200\ R & + & 68100\ G & + & 175800\ B \\ 0 & 1000\ R & + & 1000\ G & + & 1000\ B \\ -1 & 17{,}7\ R & + & 15{,}7\ G & + & 6{,}3\ B \end{array}$$

If we represent the colour contained in a total quantity of light of 1000 by the quantities R, G, B and the brightness by magnitudes, we shall find for

$b' = +1$ $\quad Col. = 221\ R + 218\ G + 562\ B$ $\quad Br. = +4{,}6\ Mg.$
$b' = -1$ $\quad Col. = 445\ R + 396\ G + 160\ B$ $\quad Br. = -4{,}4\ Mg.$

Thus the first colour may be described as a mixture of 654 white and 347 of a blue consisting of 3 R and 344 B, hence corresponding in tint to λ 466; the second colour is a mixture of 480 white and 521 of a yellow consisting of 285 R and 236 G, hence corresponding to the wavelength λ 587. A degree of glowing $b' = -2$, corresponding in colour almost with the light of petroleum, involves a decrease in brightness of 8,6 magnitudes.

For the calculation of the atmospheric absorption we have assumed that the general absorption in a gas is inversely proportional to the fourth power of the wavelength. For a layer of gas adopted arbitrarily, which after a comparison with MÜLLER's spectral-photometric measurements appeared to correspond to 1,05 atmosphere, we have calculated $f(\lambda)$ and thence found for the remaining quantity of light, the initial quantity being $1000\ R + 1000\ G + 1000\ B$:

$$783\ R + 771\ G + 571\ B,$$

or reduced to 1000 as the sum,

$$368\ R + 363\ G + 269\ B;$$

the brightness is then 0,783 of the original brightness or is diminished by 0,27 magn.

The colouring due to the absorption by 1.05 atmospheres is almost equal to that brought about by a diminishing of the degree of glowing of $^1/_3$. For the latter yields

$$257\ R + 248\ G + 184\ B$$

hence when reduced to a sum of 1000

$$372\ R + 361\ G + 267\ B$$

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[1]) In the paper read at Dusseldorf (see note p. 292) wrong temperatures are given because the difference between b and b' was overlooked. The temperatures 16000°, 7500°, 5000°, 3750°, 3000° C do not differ inter se 1, but only 0.43 in degree of glowing.

which is nearly identical with the value above. Here, however, the brightness is diminished to 0.257 of the original, hence by 1.48 magnitude.

Therefore it appears here that these two different causes produce similar colours, but that they correspond to an entirely different decrease of brightness. When comparing the two we may say that atmospheric absorption is more apt to redden, a decrease of temperature more apt to fade the light. Therefore it is impossible to derive the radiating power from the colour only, as we do not know to what degree each of the two influences, temperature and absorption, is at work in the different spectral classes. Perhaps that one day accurate spectral-photometric measurements will enable us to separate the two influences, for they give a different distribution of intensity over the spectrum. For the log. of the brightness of different λ with regard to λ 500 we find

	$\lambda = 650$	600	550	500	450	400
with abs. 1.05 atm.	+ 0.114	+ 0.083	+ 0.051	0.000	−0 084	−0.231
with glowing — $^1/_8$	+ 0.154	+ 0.111	+ 0.061	0.000	−0.074	−.0166

For the latter the decrease in intensity from the red to the violet is more regular, for the former the decrease is slower for the greater and more rapid for the smaller wave-lengths.

These calculations show that it is not strictly true that, as has been said in the preceding paper, a redder colour must necessarily involve a smaller radiating power. Where we have two influences which in different ways bear on the colour and the brightness, the possibility exists that a redder colour may be accompanied by a greater radiating power, namely when one source of light has a much higher temperature and at the same time a greater atmospheric absorption than the other. An increase of the degree of glowing of + $^1/_8$ combined with an absorption of 2 atmospheres gives such a case according to the figures given above.

Herein we have therefore a new possibility to account for the peculiarities found in the *K* stars, namely by assuming that, as compared with the *G* stars, they have a much higher temperature, which causes a stronger radiation, and which by very strong atmospheric absorption, is only little faded but greatly reddened. We must add, however, that this explanation seems little probable to us as the band-absorption, which begins at the *K*-stars and which is characteristic for the *M* stars (the 3rd type) indicates a lower temperature.

Chemistry. — *"Action of Potassium hypochlorite on Cinnamide".* By Dr. R. A. WEERMAN (Communicated by Prof. HOOGEWERFF).

(Communicated in the meeting of September 29, 1906).

From the experiments of BAUCKE [1]) on propiolamide and those of FREUNDLER [2]), VAN LINGE [3]) and JEFFREYS [4]) on cinnamide it appears that in the case of these unsaturated acids, the HOFMANN reaction to prepare an amine from an amide by means of a halogen and an alkali does not succeed.

As to the non success we may form two hypotheses: first of all that the double bond [5]) prevents the intramolecular rearrangement of atoms which must be assumed in the HOFMANN reaction, and secondly that the amine supposed to be formed, in this case $C_6H_5CH = CH\ NH_2$, suffers decomposition under the said circumstances. [6]).

The first, however, is *not* the case as from cinnamide may be prepared the urea derivative:

$$\begin{array}{l} C_6H_5C^H = C^H - N^H \\ \qquad\qquad\qquad\qquad\quad \diagdown \\ \qquad\qquad\qquad\qquad\qquad\quad CO \\ \qquad\qquad\qquad\qquad\quad \diagup \\ C_6H_5C_H = C_H - C_O - N_H \end{array}$$

where consequently one-half of the amide has undergone the transformation.

This being a case of an unsaturated amide, it is necessary to make use of the modification proposed by HOOGEWERFF and VAN DORP and not to work with free halogen. Further the hypochlorite solution must not contain any free alkali; on account of the insolubility of cinnamide and the consequent inertness, an alcoholic solution is employed.

Although at first sight it appears strange that in alcoholic solution the urea derivative is formed and not the urethane, this may be explained by the experiments of STIEGLITZ and EARLE [7]), which show that isocyanates react very readily with halogen-amides [8]).

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1) Rec. 15, 123.

2) BULL [3] 17, 420.

3) Dissertation VAN LINGE, Bazel 1896.

4) Am. Chem. Journ. 22, 43.

5) On account of the great analogy existing between the LOSSEN transformation of hydroxamic acids and the HOFMANN reaction, this first supposition was not very probable, as THIELE had prepared from the acylated cinnamohydroxamic acid the urethane $C_6H_5C^H = C^H - N^H - CO_0C_2H_5$. A second indication, though less conclusive, in the more distant analogy between the BECKMANN rearrangement and the HOFMANN reaction was the formation of isochinolin from the oxime of cinnamaldehyde. (Ber. 27, 1954).

6) THIELE, Ann. 309, 197.

7) Am. Chem. Journ. 30, 412. C 1904, I, 239.

8) This is the reason why, in the preparation of urethanes according to JEFFREYS, the sodium ethoxide should be added all at once.

In order to prepare the urea derivative, the cinnamide is dissolved in eight times its weight of 96 pCt. alcohol, and when cooled to the temperature of the room the hypochlorite solution, prepared according to GRAEBE [1]), is slowly dropped in, the free alkali being neutralised with 2N hydrochloric acid immediately before use. For every 2 mols. of amide, 1 mol. of potassium hypochlorite should be added. The liquid gets warm, and very soon a crystalline mass composed of very slender needles is deposited. After a few hours the mass is collected at the pump; this does not go very readily on account of the fine state of division. The yellowish mass is treated with hot alcohol and then washed with water. A fairly pure urea derivative is thus obtained (m. p. about 218). By recrystallisation once or twice from glacial acetic acid it is obtained pure in needles (m. p. 225—226).

0,1733 grm. yielded 0,0894 grm. H_2O and 0,4682 grm. CO_2
0,1654 „ „ 0,1863 „ „ „ 0,4467 „ „
0,1654 „ „ 13,9 CC.N at 19½° and 765 m.M.

Found	73,68	pCt. C	5,78 pCt. H	9,70 pCt. N
	73,66		5,85	
Theory $C_{18}H_{16}N_2O_2$:	73,95 pCt. C		5,51 pCt. H	9,59 pCt. N

The compound is insoluble at a low temperature in water, ligroin, alcohol, methyl alcohol, ether, carbon disulphide and benzene; at the boiling temperature slightly soluble in alcohol and benzene and freely so in glacial acetic acid, chloroform and acetone. It is insoluble in alkalis or acids.

Chemical Laboratory, Technical High School, Delft.

Astronomy. "*Mutual occultations and eclipses of the satellites of Jupiter in* 1908. By Prof. J. A. C. OUDEMANS.

(Communicated in the meeting of September 29, 1906).

N.B. In the present communication the four satellites of Jupiter, known since 1608, have been denoted by I, II, III and IV in accordance with their mean distances from the planet. The further letters *n* and *f* indicate whether the satellite is *near* or *far*, i.e. whether it is in that half of the orbit which is nearest to or furthest from the Earth. The jovicentric longitudes as well as the geocentric amplitudes are counted in "signs" and "degrees", the latter beginning from the superior

[1]) Ber. 35, 2753.

geocentric conjunction. Eastern elongation, denoted by $e.e$, has an amplitude of 3^s, western elongation, $w.e$, one of 9^s.

Not to interrupt the text unnecessarily, all particulars have found a place at the end of the paper.

FIRST PART. OCCULTATIONS.

In the numbers 3846 and 3857 of the *Astronomische Nachrichten* we find two communications relative to observations of the occultation of one satellite of Jupiter by another. The first (1) is by Mr. PH. FAUTH at Landstuhl, dated 8 December 1902, with postscripts of 29 December 1902 and 14 January 1903. The other (2) by Mr. A. A. NIJLAND at Utrecht, dated 27 February 1903.

FAUTH notes in addition that HOUZEAU, in his *Vademecum*, p. 666 mentions a couple of similar observations (3), and further that STANLEY WILLIAMS, on the 27th March 1885 at $12^h\ 20^m$, saw the third satellite pass the first in such a way that the two satellites combined had a pear-shaped appearance. (4)

The satellites of Jupiter move in orbits but little inclined to the plane of Jupiter's equator. LAPLACE assumed a fixed plane for each satellite; the plane of the satellite's orbit has a constant inclination on this fixed plane, whereas the line of intersection, the line of the nodes, has a slow retrograde motion. The inclinations of the fixed planes on the plane of Jupiter's equator amount only to a few minutes; their intersection with the plane of Jupiter's orbit is identical with the line of the nodes of the equator. The value generally adopted for the inclination of the latter plane on the orbit of Jupiter is $3°4'$, whereas the longitude of the ascending node, which therefore is also that of the fixed planes, is at present about $315\frac{1}{2}°$.

In order to be able to assign the time at which, as seen from the Earth, an occultation of one satellite by another is possible, it is necessary to know the longitude of the ascending node and the inclination of the mean fixed plane on the orbit of the Earth. At the time that the mean fixed plane, prolonged, passes through the Earth, occultations of one satellite by another may be observed. As Jupiter completes a revolution around the sun in nearly 12 years, these times will succeed each other after periods of six years. Jupiter will pass alternately through the ascending and the descending node of the plane which passes through the centre of the sun parallel to the mean fixed plane.

It follows that, as occultations of one satellite by another have been observed in 1902, we must expect that these phenomena will be again visible in 1908 (5).

To facilitate these observations I thought it desirable to calculate in advance the conjunctions of any two satellites for the most favourable part of 1908.

We have to consider that while formerly the orbits of the satellites were determined by repeatedly measuring the distances and their angles of position relatively to the planet, this method is now replaced by the measurement of the distances and the angles of position of the satellites relative to each other (especially with the heliometer) (6). For observations during a moderate interval the periodic times of the satellites may be assumed to be accurately known. Admitting this, if, leaving out of consideration KEPPLERS third law, we introduce the major axis of each satellite as an unknown quantity, the total number of such unknowns will be six for each orbit at a determined time. If, as was done by BESSEL at Königsberg in 1834—39, and by SCHUR at Göttingen in 1874—1880, the distance and the angle of position between the planet and the satellite are measured, we get two equations with six unknown quantities. If however we measure the distance and the angle of position of two satellites relative to each other, the number of unknown quantities in these equations is doubled and thus becomes 12. If finally all the combinations two by two, are observed, as was done by GILL and FINLAY at the Observatory of the Cape, we get a great number of equations with a total of 24 unknown quantities. These equations must then be solved by the method of least squares. This number becomes 29 if we add the masses of the satellites, (only to be found by the perturbations caused by one satellite in the motion of the others,) and the compression of Jupiter (7), given by the retrogradation of the lines of the Nodes on the fixed planes.

Now the observation of an occultation, even of a conjunction without an occultation, can be made by everybody possessing a telescope of sufficient power. Such an observation also furnishes two equations between the unknown quantities, at least if, for a non central occultation or a simple conjunction, the difference in latitude is measured at the filar micrometer. This consideration engaged me to compute in advance the time of these conjunctions for the most favourable part of 1908. If by experience we find that this preliminary work leads to valuable results, it might be worth while to continue it for some future period, for instance for 1914.

For the moment at which the mean fixed plane passes through the centre of the Earth, I find, 1908 July 8, $19^h,6$ Mean Time at Greenwich, (5).

This date, it is to be regretted, is very unfavourable. For on that

day Jupiter culminates at Greenwich at 2^h10^m M. T., its declination being 16°48′·5 North, whereas the Sun's declination is 22°30′ North. From these data I find for the 8th of July, for Utrecht, duly making allowance for refraction:

Setting of the upper limb of the sun at 8^h20^m mean time,
„ „ Jupiter „ 9 44 ·5 „ „ .

So there is but a poor chance for an observation of the computed occultation at Utrecht. For southern observatories it is somewhat better. At the Cape for instance, we have:

Sunset at 5^h 5^m mean time,
Setting of Jupiter „ 7 25 „ „ .

We thus find that on July 8, 1908, at Utrecht, the setting of the sun precedes that of Jupiter by $1^h24^m·5$; at the Cape by 2^h20^m.

We have computed all the conjunctions of the satellites of Jupiter which will occur between 31 May and 20 July 1908. In what follows a short account is given of the way which led to our results.

In the *Nautical Almanac* are given the *Geocentric Superior Conjunctions*; in the Almanac of 1908 they will be found on pp. 504, 505.

To begin with, a separate drawing was made of the four orbits, which were supposed to be circular, for each interval of two periods of I (about 85^h). On these orbits we plotted the positions of the satellites for each second hour, making use of divided pasteboard arcs. The number of hours elapsed since the moment chosen as a starting-point were noted for each position. The equation of the centre etc. was neglected.

The scale of this drawing gave 4″ to 1 mm. The radii, of the orbits therefore were: for I 27·9 mm.; for II 44·45 mm.; for III 70·9 mm. and for IV 124·7 mm.

The direction from the Zero of I to the common centre of all the circles showed the direction towards the Earth. Knowing this, we could easily find for each of the six possible combinations of two of the satellites, those equal hour numbers, the connecting line of which is parallel to this direction.

These connecting lines show the approximate times at which, as seen from the Earth, one of the satellites is in conjunction with another. The want of parallelism of the real lines joining the Earth with the satellites, in different parts of their orbits, may safely be disregarded. The plate annexed to this paper represents, reduced to half the scale, the drawing for the period of 85 hours, following 12 July 1908, $11^h2^m·3$ M. T. Greenwich.

The dotted lines indicate the lines connecting the equal numbers.

Each of them represents a conjunction of two satellites. The corresponding hours read off from the figure are :

$6^u{\cdot}2$	:	IV_f	occulted by	III_n,
21 ·8	:	IV_f	„ „	I_n,
25 ·0	:	IV_f	„ „	II_n,
35 ·0	:	I_f	„ „	II_n,
66 ·25	:	II_f	„ „	I_n,
71 ·0	:	III_f	„ „	I_n,

They were added to the instant which must be regarded as the startingpoint for this figure. The instants of the conjunctions were next converted into civil time of Paris by the addition of $12^h9^m21^s$. The elongation and the latitude of both the satellites, expressed in radii of Jupiter, were then computed by the aid of the *Tables écliptiques* of Damoiseau, 2nd part. (8). In the case that the elongations did not perfectly agree, a slight computation led to a more accurate result for the time of conjunction (9).

In the case that the two satellites moved apparently in opposite directions, (which happens if the one is in the further part of its orbit, the other in the nearer part), the correction to the adopted time was mostly insignificant.

If, on the contrary, they moved the same way (which happens if both are "far" or if both are "near", so that the one has to overtake the other) the correction amounted sometimes to an hour or more. In every case, in which the correction exceeded 20 minutes, the computation was repeated with the corrected time. Further below will be found the list of the results. From May 31 to July 19, i.e. during a period of fifty days, there occur 72 conjunctions. It is to be regretted that at a determined place of observation but very few of them will be visible. For only those conjunctions are visible which occur between sunset and the setting of Jupiter. For Utrecht we have, in mean time :

	Setting of the upper limb of the Sun	Setting of Jupiter	Difference
1908 June 1	8^h10^m	11^h54^m	3^h44^m
„ 11	8 20 ·5	11 19	2 58 ·5
„ 21	8 24	10 44	2 20
July 1	8 24	10 9	1 45
„ 11	8 18	9 34	1 16
„ 21	8 7 ·5	8 59	0 51 ·5

For the Cape of Good Hope:

1908	June 1	4^h59^m	9^h18^m	4^u19^m
	„ 11	4 57 ·5	8 46 ·5	3 49
	„ 21	4 58	8 16	3 18
	July 1	5 2	7 46	2 44
	„ 11	5 6 ·5	7 16	2 9 ·5
	„ 21	5 13	6 47	1 34

The circumstances are thus seen to be considerably more favourable for a southern than for a northern observatory.

Several of the occultations will not be visible because the common elongation falls short of unity i. e. of the radius of Jupiter. This is the case of Nos. 8, 9, 12, 13, 15, 16, 20, 23, 39 and 64. In the first eight of these cases and in the last one the planet stands between the two satellites. In case No. 39 both the satellites I and IV are covered by the planet [1]).

For other conjunctions it may happen that one of the satellites is invisible because of its being in the shadow of the planet. Such cases are:

(N°. 21),	June 13	9^h28^m	M. T. Grw.,	II	eclipsed,
(N°. 31),	„ 20	12 51	„ „	II	„ ,
(N°. 51),	July 4	18 15 ·7	„ „	II	„ ,
(N°. 65),	„ 13	12 3	„ „	IV	„ .

If the satellite which at the conjunction is nearest to the Earth is eclipsed by the planet's shadow, it might, as seen from our standpoint, project itself wholly or partially as a black spot on the other satellite. The case however has not presented itself in our computations.

Possibly the last of the conjunctions just mentioned may really be visible; for according to the *N. Almanac*, the reappearance of IV from the shadow of the planet takes place at $12^h7^m15^s$ M. T. Greenwich and the predicted eclipses of this satellite are occasionally a few minutes in error. A few minutes later, according to the *N. Almanac* at 12^h16^m, II enters the disc of Jupiter.

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[1]) According to the Nautical Almanac we have for this night (M. T. of Greenwich):

IV. Occultation Disappearance 10^h19^m,
I. Occultation Disappearance 11 20,
I. Eclipse Reappearance 14 26 27^s,
IV. Occultation Reappearance 15 13,
IV. Eclipse Disappearance 18 5 6,
IV. Eclipse Reappearance 22 52 2.

NOTES.

(1) The article of FAUTH, abridged, runs thus:

— — — Ausser den in HOUZEAU, Vademecum p. 666 aufgeführten Beobachtungen, (vid. below Note 3), kenne ich aus neuerer Zeit nur einen Fall: STANLEY WILLIAMS sah am 27 März 1885 an einem 7 cm. Rohre mit 102-facher Vergrösserung um $12^h\ 20^m$ den III Trabanten vor dem I, wobei beide ein birnformiges Objekt bildeten.

— — — In fünf Wochen konnte ich drei Bedeckungen verfolgen, wobei auzunehmen ist, dass mir durch schlechte Witterung etwa 10 andere Gelegenheiten entgangen sein mögen, unter denen sicher einige Bedeckungen vorkommen. Nach meiner Erfahrung können Konjunctionen der Jupitermonde unter sich weit genauer beobachtet werden als Bedeckungen durch Jupiter oder Vorübergänge vor ihm. Somit möchten die hier angegebenen Beispiele Anlass bieten, in den späteren Oppositionen Jupiters den durchaus nicht seltenen Bedeckungen oder wenigstens Berührungen und sehr nahen Konjunktionen der Trabanten unter sich mehr Aufmerksamkeit zu schenken, zumal schon kleine Instrumente zur Wahrnehmung der Phasen einer event. Bedeckung genügen. Die Beobachtungen der letzten Zeit sind:

1. Oct. 7; II bedeckt I; die S. Ränder berühren sich und I ragt im N. etwas hervor. Konj. um $9^h\ 16^m$ M. E. Z. [1])

2. Oct. 23; II bedeckt III so, dass die Mitte von II nördlich am N. Rand von III vorbeigeht; Konjunktion um $8^h\ 7^m\ 3^s{,}5$.

3. Nov. 10; III bedeckt I so, dass der S. Rand von III die Mitte von I streift (gute Luft); Konjunktion um $7^h\ 33^m\ 20^s$.

Instrument: 178 mm., Vergrösserung 178 fach.

Landstuhl, 1902 Dez. 8.

P.S. vom 29 Dezember. Am Abend des 24 Dezember gelang nochmals die Beobachtung einer Bedeckung, bei welcher I über IV hinwegzog. Aus je fünf vor- und nachher notierten Zeitmomenten folgen als Mittelwerte $6^h\ 24^m{,}25$, $24^m{,}625$, $24^m{,}50$, $24^m{,}625$ und $24^m{,}50$. Die Konjunktion fand also statt $6^h\ 24^m\ 30^s$.

Der Uhrstand war um 3^h mit dem Zeitsignal verglichen worden. IV Stand ein wenig südlicher als I, vielleicht um ein Viertel seines Durchmessers. Die weitaus interessantere Konjunction zwischen II und IV am 25 Dezember blieb gegenstandslos, weil IV um etwa zwei Durchmesser vorüberging,

P.S. vom 14 Januar (1903). Heute Abend, am 14 Januar, bewegte

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[1]) i. e. Mittlere Europäische Zeit, 1^h later than Greenwich-time.

sich der Trabant III über II hinweg. Die sehr schlechte Luft liess nur den ersten Kontakt auf etwa $6^h\ 2^m$ feststellen. Um $6^h\ 18^m$ mochten sich beide Komponenten so weit getrennt haben, dass dies in einem weniger schlechten Augenblick bemerkt wurde; um $6^h\ 32^m$, dem nächsten blickweisen Auftauchen der beiden Lichtpunkte, waren diese um etwa einen Durchmesser von einander entfernt. Die Bedeckung war fast genau central. PH. F.

(2) Mr. NIJLAND writes in N°. 3857 of the *Astronomische Nachrichten*: — — — Am 15 Juli 1902 fand eine Konjunktion der Trabanten II und III statt, welche ich bei guter Luft am Refraktor (Brennweite 319 cm., Oeffnung 26 cm.) mit Vergr. 248 beobachten konnte. Es wurde III nahezu central von II bedeckt. Einige Minuten lang blieb eine feine schwarze Linie zwischen den beiden Scheibchen sichtbar, welche um $14^h10^m11^s$ M.Z. Utrecht verschwand and um $14^h20^m31_s$ wieder erschien; die Konjunktion musz also um $14^h15^m21^s$ stattgefunden haben. Dass diese Trennungslinie vor und nach der Konjunktion immer dieselbe Richtung hatte, und zwar scheinbar senkrecht auf der Bahnebene der Trabanten stand, mag als Beweis dafür gelten, dass der Vorübergang wirklich nahezu central gewesen ist. Dann lässt sich aber aus dieser centralen Passage die Summe der Durchmesser der Monde II und III mit erheblicher Genauigkeit bestimmen.

Nehme ich für die mittlere Entfernung ♃ — ☉ die Halbmesser der Bahnen gleich 177″,8 und 283″,6, so finde ich für die relative Bewegung von II und III zur Beobachtungszeit 13″,86 pro Stunde. Aus der beobachteten Zeitdauer von $10^m20^s = 0^h172$ folgt dann für die Summe der beiden Durchmesser, 2″,38. Wird (siehe die Angaben von DOUGLASS, Astr. Nachr. 3500) für das Verhältniss der Durchmesser von II und III $^4/_{11}$ angenommen, so finde ich, in vorzüglicher Uebereinstimmung mit den a. a. O. genannten Werten, für den Durchmesser von II 0″,87 und von III 1″,51 (in mittl. Entf.).

Utrecht, 1903 Febr., 27. A. A. NIJLAND.

Remark. As from the observed instants I derived a result slightly different from that of Mr. NIJLAND, this gentleman allowed me to consult his reduction of the observation. It appeared that, in order to find the amplitudes, he had combined the preceding geocentric superior conjunction with the following transit, from the ingress and egress of which the inferior conjunction could be derived. A slight error had however been committed in the computation. After correction the relative motion of the two satellites was found to be 13″·786 and the sum of the diameters 2″·374. Moreover their proportion was, evidently erroneously and

against the real intention. put at 4 to 11 instead of at 4 to 7. We thus get for the diameters 0″·863 and 1″·511, which is still in good agreement with the result of Mr. NIJLAND. As values have been assumed for the radii of the orbits which hold for the mean distance of Jupiter from the sun, these values need no further reduction.

(3) We find in HOUZEAU, *Vademecum* (Bruxelles, 1882), p. 666: On rapporte une occultation du satellite II par le satellite III, observée à Sommerfeld, près de Leipzig, par C. ARNOLDT, le 1er novembre 1693, (WHISTON, *The longitude discovered by the eclipses*, 8°, London, 1738), et une autre du satellite IV, également par le IIIme, vue par LUTHMER à Hanovre, le 30 octobre 1822 (*Nature*, 4°, London; vol. XVII, 1877, p. 148).

1st Remark. The little book of WHISTON here quoted is in the library of the University at Utrecht, Division P, 8vo, number 602. We have turned over the leaves several times, but have not found any mention of the observation of C. ARNOLDT. It is true that the author, in § XVIII, recommends the observation of the mutual occultations of the satellites. He remarks that, if at such an occultation they have opposite motions, the relative velocity is "doubled". He mentions the complaint of DERHAM [1]), that the strong light of Jupiter renders the observation of these occultations rather difficult. He remarks that, the interval being equal, their number must be one and a half time as large as that of the eclipses. Again he mentions that LYNN is the first who, in the *Philosophical Transactions* N°. 393, has proposed to apply these conjunctions to the determination of the longitude, seeing that they can often be observed with an accuracy of less then half a minute [2]). But I do not find the observation of a single occultation nor its prediction.

It needs hardly be said that the conjunctions, visible from places, the difference in longitude of which is to be determined, are too rare to be of much importance for the purpose. In accuracy of observation they are at all events surpassed by occultations of stars. But they may well be compared with the eclipses of the satellites of Jupiter and are indeed superior to them in this respect that they yield a result in a few minutes which is independent of the optical power of the telescope. For the eclipses this is only true in the case of the combination of a disappearance with a reappearance.

2nd Remark. The original account of the observation of LUTHMER was communicated by him to BODE who inserted it in the (Berliner) *Astronomisches Jahrbuch* für 1826, p. 224:

"Am 30 Oct. Ab. 6u 55′ Bedeckung des vierten ⅣTrabanten vom dritten."

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[1]) POGGENDORFF's *Biographisches Wörterbuch*, (article W. DERHAM) gives no reference to the passage where this complaint is to be found, nor even to any paper on the observation of the satellites of Jupiter.

[2]) At least if there were no undulation of the images. See at the end of note 4

If we assume $9°42' = 38^m48^s$ East of Greenwich for the longitude of Hannover, this is $= 6^h\ 16^m\ 12^s$ M. T. of Greenwich, at least supposing that at that time it was already usual to give the observations expressed in mean time.

In *Nature*, XVII (Nov. 1877—April 1878) p. 149 (not 148) we find in "*Our Astronomical Column*":

"JUPITER'S SATELLITES. — Amongst the recorded phenomena connected with the motions of the satellites of Jupiter are several notices of observed occultations of one satellite by another, and of small stars by one or other of the satellites. [1]) The following cases may be mentioned: — On the night of November 1, 1693, CHRISTOPH ARNOLDT, of Sommerfeld, near Leipzig, observed an occultation of the second satellite by the third at $10^h\ 47^m$ apparent time. On October 30, 1822, LUTHMER, of Hannover, witnessed an occultation of the fourth satellite by the third at $6^h\ 55^m$ mean time.

It thus appears that the editor of *Nature* also took it for granted that the statement must be understood to have been made in mean time.

(4) I did not succeed in finding the account of this observation of STANLEY WILLIAMS in any of the journals accessible to me, and therefore applied to the author, who lives at Hove near Brighton, for particulars about the place of its publication.

He kindly replied on the 7th instant, that the details of his observation of 27 March 1885 were published both in the 41th volume of the "*English Mechanic*" and in the volume for 1885 of the German Journal "*Sirius*".

He had moreover the courtesy of communicating to me the original account of the observation in question. From this account the following passages may be quoted:

Occultation of satellite I by satellite III.

1885 March 27, ... $2^3/_4$ inch refractor. Power 102.

11^h55^m (Greenwich mean time). They are now only *just* free from contact. $_{III}\bigcirc\!\!\!\!\bigcirc_{I}$, like an elongated star with little more than a black line between the components.

12^h00^m to 12^h04^m. After steady gazing I cannot see any certain separation between the satellites, and therefore with this instrument and power first contact must have occurred about 12^h02^m. Definition is very bad, however, and in a larger telescope there probably might still be a small separation between the limbs.

[1]) It is to be regretted that these "several notices of observed occultations of one satellite by another" are not more fully quoted.

12^h10^m. They now appear as one elongated satellite. At times a trace of the notches is apparent.

12^h20^m. The elongation is now very nearly at right angles to the direction of the motion of the satellites, and is so slight as to be scarcely noticeable in this bad and unsteady definition. I think from the smallness of the elongation that nearly half satellite I must be concealed behind III. In this bad definition it is not possible to say which satellite is in front of the other from the appearance alone.

In his letter Mr. STANLEY WILLIAMS mentions the remarkable fact that he too observed on 15 July 1902 the same conjunction which has been described by NIJLAND. His instrument was a reflector of $6^1/_2$ inch, with a power of 225. The following are the particulars as communicated:

1902 July 15, $13^h45^m{\cdot}2$. Satellites II and III are in contact. The one will occult the other. See diagram .

13^h52^m. The satellites form one disc, which has the slightest possible elongation in a north and south direction. Owing to confused seeing this disc always appeared more or less fuzzy, and it is impossible from the appearance alone to say, which satellite is occulting the other.

13^h56^m. The combined disc is considerably elongated now.

$14^h02^m{\cdot}2$. Satellites II and III in contact as in diagram adjoining .

14^h04^m. Satellites clearly separated. The occultation must have been nearly central. II is a little more south now relative to III, than it was before occultation. Possibly the slight elongation noted at 13^h52^m was not real.

The above times are Greenwich mean times. Satellite III was on the farther side of its orbit moving east, II on the near side moving west. As the disc of III is larger than that of II, the phenomenon should be described as a transit of II over or across III, rather than an occultation of one satellite by the other.

The arithmetical mean of $13^h45^m{\cdot}2$ and $14^h2^m{\cdot}2$ is $13^h53^m{\cdot}7$, which is $1^m{\cdot}1$ earlier than NIJLAND's observation.

(5) For the numbers which follow we refer to KAISER's "*Sterrenhemel*", 4th Edition, p. 707 and following.

In the 4th Vol. of his *Mécanique Céleste*, p. 62, TISSERAND, following SOUILLART, adopts inclinations for the orbits of III and IV, which

respectively excéed those given in the "*Sterrenhemel*" by + 4″ and — 8″.

According to LEVERRIER we have, for the orbit of Jupiter in 1908,0:

Ascending Node = 99°31′56″,

Inclination = 1 18 29 .

The fixed plane of the first satellite coincides with the plane of Jupiter's equator: the longitude of the ascending node on the plane of Jupiter's orbit, for the beginning of 1908 is therefore 315°33′35″, the inclination 3° 4′ 9″.

Furthermore we have for the four fixed planes relative to the plane of Jupiter's orbit:

	Long. asc. node	Inclination	
I	315°33′35″	3° 4′ 3″	Epoch 1908.0.
II	315 33 35	3 3 4	
III	315 33 35	2 59 11	
IV	315 33 35	2 39 57	

For the mean fixed plane of the three first satellites we thus find: longitude of ascending node on the plane of Jupiter's orbit at the beginning of 1908: 315°33′35″, inclination 3° 2′ 6″.

Moreover we have for the respective fixed planes in 1908, according to TISSERAND:

	long. asc, node	Change in 1000. days	Inclination
II	122°·293	— 33°·031	0°28′ 9″
III	26 ·173	— 6 ·955	0 10 44
IV	238 ·982	— 1 ·856	0 13 51

The effect of these inclinations, however, is but trifling. At the distance of 90° from the node they produce only deviations

for II of 1″·46,

„ III „ 0″·89,

„ IV „ 2 ·01.

The determination of the position of the fixed planes, as also that of the planes of the orbits of the satellites relative to these, will be much improved by the measurements which DE SITTER at Groningen is making on photographic plates. Eventual observations of conjunctions of the satellites, rather even of occultations, will contribute their part in this determination and will furnish a test for the adopted values.

In the meeting of our section of last March a provisional account of these measures by DE SITTER was communicated by Messrs J. C. KAPTEYN and E. F. VAN DE SANDE BAKHUYZEN [1]).

Our computations were then already too far advanced to keep them back altogether; but we hope that by the side of these measures they still may have their use, for this reason that conjunctions and mutual occultations of the satellites may well be observed at several observatories which are not equipped for taking photographs.

From the preceding numbers we find for the position of the fixed plane relative to the ecliptic (for 1908,0).

$$\text{Ascending Node } 336°48'23'' = \Omega,$$

$$\text{Inclination } \quad 2\ \ 7\,17 = \mathrm{I},$$

Now, if $R_{♃}$, $L_{♃}$ and β represent the radius vector, the longitude and the latitude of Jupiter; $R_{♁}$, $L_{♁}$ the radius vector and the longitude of the Earth, (those given in the N. Almanac after correction for aberration), the condition that the fixed plane must pass through the Earth is expressed by:

$$R_{♃} \cos\beta \sin(L_{♃} - \Omega) - R_{♃} \sin\beta \cot I = R_{♁} \sin(L_{♁} - \Omega),$$

which is satisfied July 8, 1908 at $19^h38^m{\cdot}3$. For at that moment

$$\log R_{♃} = 0{\cdot}728527 \qquad \log R_{♁} = 0{\cdot}007179$$

$$L_{♃} = \quad 141°23'\ 9''{\cdot}0 \qquad L_{♁} = 286°40'\ 3''{\cdot}5$$

$$\beta \ = + \quad 0\ 52\,26\ {\cdot}73 \qquad \Omega \ = 336\ 48\,52\ {\cdot}0$$

so that our equation becomes

$$1{\cdot}423706 - 2{\cdot}204190 = -0{\cdot}780484$$

Similarly we find for the instant at which the same plane passes through the centre of the sun:

25 April 1908 at $18^u{\cdot}5$ M. T. Grw.

On both sides of this latter epoch there exists the possibility of an eclipse of one satellite by another, at the time of the heliocentric conjunctions. We hope to treat this subject in the second part of this communication.

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1) This provisional account may be considered as a sequel to the thesis of Mr. DE SITTER. This thesis, maintained by him at Groningen on 17 May 1901, bears the title: *Discussion of Heliometer-observations of Jupiter's satellites made bysir* DAVID GILL *K.C.B. and* W. H. FINLAY *M.A.* Further particulars will be given in the *Annals of the Royal Observatory at the Cape of Good Hope.*

(6) In 1833—39 BESSEL, at the Heliometer, measured not only distances of all the satellites from both limbs of the planet, but also angles of position of the centre of the planet to III and IV.

His heliometer was the first big instrument of the sort made in the establishment of FRAUNHOFER; the objective had an aperture of 70·2 Par. lines and a focal distance of 1131·4 Par. lines = 7 feet 10 inches 3·4 lines, Paris measure, (15·84 and 255·22 c.M.). The mean error of a single observation of distance (which properly was the mean of eight pointings) appeared to be

for I	± 0″26,	for the mean distance resulting from all the measures,	± 0″055
„ II	± 0·24,	„ „ „ „ „ „ „ „ „	± 0·067
„ III	± 0·31,	„ „ „ „ „ „ „ „ „	± 0·042
„ IV	± 0·43,	„ „ „ „ „ „ „ „ „	± 0·045
Mean.	± 0·31,	„ „ „ „ „ „ „ „ „	± 0·052

SCHUR, at Göttingen, used the heliometers which were made by MERZ at München for the observation of the transits of Venus in 1874 and 1882. The aperture of the objectives of these instruments was 34 Par. lines, something less than half that of the heliometer of Königsberg; the focal distance was 3½ feet (113·7 cM.).

At these heliometers the reading, instead of being made on the drums of two micrometers, was made by a microscope at right angles to two scales fitted to the two halves of the objective. As however in this way more time was required than for reading the druns of a micrometer of BESSEL's instrument, SCHUR, instead of taking the mean of eight pointings, was content with the mean of four pointings, which also make a complete measurement.

The mean errors of each observation obtained by SCHUR for a complete set of four measures was:

for	I ± 0″·34,
„	II ± 0 ·44,
„	III ± 0 ·37,
„	IV ± 0 ·42,
Mean:	± 0″·39,

a result, which, taking into account the shorter focal distance, may be considered fairly good. BESSEL as well as SCHUR aimed not so much at the determination of the position of the orbits of the satellites as at that of the mass of Jupiter.

SCHUR improved in different respects the reduction of the observations of the measures made by BESSEL. In consequence, the mean errors of the single determinations of BESSEL were considerably lessened. The numbers quoted just now, became:

for I $\pm$ 0″·21,
„ II $\pm$ 0 ·10,
„ III $\pm$ 0 ·26,
„ IV $\pm$ 0 ·30,
Mean: $\pm$ 0″·24.

As has been mentioned already, GILL and FINLAY, acting on a suggestion formerly made by OTTO STRUVE [1]), did not measure the distances and the angles of position of the satellites relative to the centre of the planet, but relative to each other. (The instrument at their disposal, a heliometer of REPSOLD, aperture 7½ inch = 19·05 cm., focal distance somewhat over 2 Meter, far surpassed in perfection all the instruments used up to that time). These observations can be made with much more precision. The drawback is that the formation of the equations of condition and their solution become more complex and absorb much more time. Both the gentlemen named and Mr. de SITTER have not been deterred by this consideration. They found $\pm$ 0″·087, a number considerably less than that of BESSEL, for the probable error of the measurement of a single distance. Mr. de SITTER even finds that the probable error of the mean distances (the real unknown quantities) does not exceed $\pm$ 0″·020 or $\pm$ 0″·021.

(7) It may be remarked that Mr. de SITTER found it expedient to alter the choice of the unknown quantities. He retained for each satellite: the longitude in the orbit, the inclination and the ascending node relative to an adopted position of the fixed plane, but not the eccentricity nor the position of the perijovium and the mass. There thus remained as unknown quantities only three elements of each satellite. On the other hand he introduced corrections of the coefficients of the perturbations or rather of the periodic terms, which afterwards must lead to the knowledge of the mass of the satellites, to that of the eccentricities and of the position of the

[1]) *Vide* the first report of HERMANN STRUVE, in the first supplementary vol. of the Pulkowa observations, 1st page at the bottom.

apsides. He further introduced two unknown quantities, *viz.* the constant errors which might vitiate the observations of the two observers GILL and FINLAY. He thus also obtained a total of 29 unknown quantities. It need not be said that the solution of about 400 equations with so many unknown quantities, is an enormous labour. Still, owing to the help of some other computers, this labour has been brought to a happy issue.

We must not enter here into further particulars about this important work, though we did not feel justified in omitting to mention it altogether. I will only remark that it is not sufficient to determine the position of the planes of the orbits of the satellites for one epoch; for as was already remarked the position of these planes changes continually. It seems that these changes may be sufficiently represented by assuming a regular retrogradation of the line of intersection with a fixed plane, the inclination remaining the same.

The main cause of this retrogradation is the polar compression of Jupiter. It is desirable however to establish the amount of this retrogradation by the observations, and to derive afterwards the compression by means of this amount. Consequently the position of the planes of the orbits has to be determined for different epochs. In this respect too Mr. DE SITTER has done good work, *vide* the communication already mentioned, presented in the meeting of last March by Messrs KAPTEYN and E. F. VAN DE SANDE BAKHUYZEN.

(8) The same volume, which contains the ecliptic tables of DAMOISEAU, contains also in a second part (not mentioned on the title) tables "*pour trouver les configurations des satellites de Jupiter.*"

We have contemplated whether it would not be desirable not to use these tables, unmodified, for our computations. We have therefore taken note of the investigations of SOUILLART, ADAMS, MARTH, GILL, FINLAY, and de SITTER, but it appeared that such a course would aggravate our labour very considerably. We would have had to determine new elements for all the satellites and to compute new tables. This would have caused considerable retardation, unnecessary for our purpose, which was no other than to prepare astronomers for the observation of the conjunctions visible in 1908.

We therefore have based our computations on the tables of DAMOISEAU, but we have first examined in how far they represent the observed conjunctions. The following summary shows not only the difference between the observation and the tables in the elongations x and x', of the two satellites, expressed in radii of Jupiter, but also their difference in time.

Observer	Date	Occultation of	by	Error	Relative hourly motion	Corr. of Table	$y'-y$
				r	r	m	r
Fauth	1902 Oct 7	II_f	I_n	0,025	1,278	+ 1,2	+ 0,04
»	» » 23	III_f	II_n	0,02	1,130	+ 1,1	+ 0,08
»	» Nov. 10	I_f	III_n	0,00	0,883	0,0	+ 0,13
»	» Dec. 24	IV_f	I_n	0,10	1,089	— 5,5	+ 0,00
»	1903 Jan. 14	II_n	III_n	0,11	0,344	—19,2	— 0,05
Nijland	1902 July 15	III_f	II_n	0,08	0,751	+ 6,4	— 0,01
Stanley Williams	» » »	»	»	0,07	0,751	+ 5,3	— 0,01
» »	1885 March 27	I_n	III_n	0,00	0,292	0,0	+ 0,01

The observation of LUTHMER in Hannover, of Octob. 30, 1822 is not contained in this table. Its calculation yields the result:

	Jovic. Long.	Amplitude	x	y en y'
III	$10^s 26° \cdot 77$	$8^s, 22° \cdot 25$	$-15^r \cdot 21$	$+0^r \cdot 18$
IV	9 6·74	7 2·22	$-14 \cdot 40$	$-0 \cdot 74$
		Difference	$+0^r \cdot 81$	$+0^r \cdot 92$

So there is a difference in the amplitudes, of $0^r \cdot 81$, $= 0 \cdot 81 \times 18'' \cdot 37 = 14'' \cdot 9$, in the latitudes, of $0 \cdot 92 = 16'' \cdot 9$. Probably the observation has been made with an unsatisfactory instrument, for it is impossible to suppose an error of this amount in the tables of DAMOISEAU for 1822. The difference in sign of the latitudes y and y' is explained by the fact that the longitude of the ascending node of the fixed plane was $10^s 14° \cdot 37$, which is intermediate between the two jovicentric longitudes.

As the two satellites moved in the same direction, the hourly change of distance was small, *viz.* $0^r \cdot 280$. It would thus require nearly three hours to annul the difference of $0^r \cdot 81$.

The remaining conjunctions, however, show a satisfactory accuracy and we may thus expect that the table, as given below, will serve its purpose.

As a second test I have computed, by the aid of the second part of DAMOISEAU, the two superior conjunctions and the intermediate inferior conjunctions of II, and I have compared these to those given

in the *Nautical Almanac* of 1902. The epochs were found a little earlier, to wit:

superior conjunction of 10 July, $10^{h}46^{m}\cdot9$ M.T. Grw. $0^{m}\cdot7$ earlier
inferior conjunction (mean
of ingress and egress) 16 July $5^{h}40^{m}\cdot0$ „ „ $0^{m}\cdot3$ „
superior conjunction 17 July $23^{h}54^{m}\cdot4$ „ „ $0^{m}\cdot7$ „
all three less than a minute.

Now, as the conjunctions in the *Nautical Almanac* have been calculated by the aid of DAMOISEAU's *tables écliptiques* (making allowance for some slight corrections indicated by ADAMS) the differences must be solely due to the fact that in DAMOISEAU's second part the *main* terms only of the equations and perturbations have been taken into account.

The same tables represent as accurately the superior conjunction of I on January 1, 1908, $14^{h}4^{m}\cdot2$ M.T. Grw. = January 2, $2^{h}13^{m}\cdot55$ civil time of Paris; the error amounts to $0^{\circ}\cdot07$ or $0^{r}\cdot01$ linear measure only, an arc traversed by the satellite in $0^{m}\cdot5$.

(On the terms taken into account in the second part of the tables of DAMOISEAU *vide* 3rd appendix below).

In his letter Mr. STANLEY WILLIAMS mentions another rare observation, made as well by himself as by the Spanish observer J. COMAS of Valls, (near Taragona), on 14 August 1891, to wit of the coincidence and of the subsequent separation of the shadows of two satellites on the planet. He concludes that an eclipse must have taken place. These phenomena will be treated in the second part of this communication.

(9) Below follows the table which has served for this computation. The unit, the radius of Jupiter, is $18''\cdot37$. SOUILLART states that he found mentioned in the papers of DAMOISEAU that this number was borrowed from ARAGO. According to HOUZEAU, ARAGO must have made the determination by means of the double image micrometer (an invention made nearly simultaneously by himself and PEARSON; of the latter the observatory at Utrecht possesses a specimen). Particulars about these measures are not known. The number is smaller than that found by other astronomers, *vide* for instance HOUZEAU, p. 647—650; SEE, *Astron. Nachr.* N°. 3670 (15 Aug. 1900).

Hourly change of the elongation x as a function of the amplitude.

+				I		II		III		IV		—			
				r		r		r		r					
0s	0c	0s(12)	0c	0,895	4	0,708	3	0,560	2	0,420	2	6s	0c	6s	0m
	5	11	25	0,891	10	0,705	8	0,558	7	0,418	4		5	5	25
	10	11	20	0,881	17	0,697	14	0,551	10	0,414	8		10	5	20
	15	11	15	0,864	23	0,683	18	0,541	15	0,406	11		15	5	15
	20	11	10	0,841	30	0,665	23	0,526	18	0,395	14		20	5	10
	25	11	5	0,811	36	0,642	29	0,508	23	0,381	17		25	5	5
1	0	11	0	0,775	42	0,613	33	0,485	26	0,364	20	7	0	5	0
	5	10	25	0,733	48	0,580	37	0,459	30	0,344	22		5	4	25
	10	10	20	0,685	53	0,543	42	0,429	33	0,322	25		10	4	20
	15	10	15	0,632	57	0,501	46	0,396	36	0,297	27		15	4	15
	20	10	10	0,575	62	0,455	49	0,360	39	0,270	29		20	4	10
	25	10	5	0,513	66	0,406	52	0,321	41	0,241	31		25	4	5
2	0	10	0	0,447	69	0,354	55	0,280	43	0,210	32	8	0	4	0
	5	9	25	0,378	72	0,299	57	0,237	45	0,178	34		5	3	25
	10	9	20	0,306	75	0,242	59	0,192	47	0,144	35		10	3	20
	15	9	15	0,231	76	0,183	60	0,145	48	0,109	36		15	3	15
	20	9	10	0,155	77	0,123	61	0,097	48	0,073	36		20	3	10
	25	9	5	0,078	78	0,062	62	0,049	49	0,037	37		25	3	5
3	0	9	0	0,000		0,000		0,000		0,000		9	0	3	0

Finally we will give below, *vide* pp. 334 and 335, two instances of computation; one of a case in which the apparent motion of the two satellites was opposed, the other in which it was in the same direction.

1st Appendix. *What is the maximum duration of the several occultations of one satellite by another?*

We have seen above that it took $19^{m}\cdot2$ to annul the small difference of the elongations of $0^{r}\cdot11$ ($2''\cdot0$). This was caused by the minuteness of the relative motion of the satellites. But in the case that the hourly motions, which we will denote by u and u', are *absolutely* equal, the denominator of the fraction $\frac{x' - x}{u' - u}$ is *zero*.

The case then corresponds to that of the "Station of Venus" and it is a very ancient problem to compute its epochs.

Let be r and r' the radii vectores of two satellites; θ and θ' the corresponding amplitudes, then for the occultation:

$$r \sin \theta = r' \sin \theta'.$$

The condition of an equal change of longitude leads to:

$$r \cos \theta \frac{d\theta}{dt} = r' \cos \theta' \frac{d\theta'}{dt}.$$

Now, if T and T' represent the sidereal periods, we have, neglecting the apparent movement of Jupiter:

$$\frac{d\theta}{dt} : \frac{d\theta'}{dt} = \frac{1}{T} : \frac{1}{T'} = \frac{1}{r^{3/2}} : \frac{1}{r'^{3/2}},$$

consequently:

$$r^{-1/2} \cos \theta = r'^{-1/2} \cos \theta',$$

from which:

$$\cos^2 \theta = \frac{r}{r'} \cos^2 \theta' = \frac{r}{r'} - \frac{r}{r'} \sin^2 \theta'.$$

Adding

$$\sin^2 \theta = \frac{r'^2}{r^2} \sin^2 \theta',$$

we get

$$1 = \frac{r}{r'} + \left\{ \left(\frac{r'}{r} \right)^2 - \frac{r}{r'} \right\} \sin^2 \theta',$$

Therefore, putting $\frac{r'}{r} = \mu$,

$$\sin^2 \theta' = \frac{1 - \frac{1}{\mu}}{\mu^2 - \frac{1}{\mu}} = \frac{\mu - 1}{\mu^3 - 1} = \frac{1}{\mu^2 + \mu + 1}$$

and

$$\sin^2 \theta = \frac{\mu^2}{\mu^2 + \mu + 1}.$$

The equality of the hourly changes of the two elongations of course only lasts for an instant; very soon inequality sets in and the two satellites begin to separate. Meanwhile it may be long ere such becomes perceptible at the telescope, only, in a case like the present, the satellites do not pass each other, but after the conjunction they have the same position the one to the other as before.

As an example take a conjunction of I and II under the circum-

stances in question. Let the amplitudes be between 0 and 3 signs, so that both the satellites, as seen from the Earth, (the head being turned to the North Pole), are to the left of and both receding from the planet. Before the conjunction I is to the right of II, but the motion of I is quicker than that of II. I will overtake II as soon as its amplitude is 44°39′, that of II being then 26°14′. At the same time, however, the apparent velocities are equal. Now as I approaches its greatest elongation it retards its motion much more considerably than II, the amplitude of which is so much smaller. The consequence is that, after the conjunction, I is left behind, and gets again to the right of II as before conjunction.

This case represents a transition between two other cases. 1. If, under the same circumstances I is somewhat more in advance (has a greater amplitude), it will pass II, but after a while will be overtaken by II, which then, as seen from the Earth, passes behind it. 2. If, however, I is somewhat less ahead, it will continue to be seen to the right of II, the distance I — II going through a minimum but not reaching zero.

Now, in order to answer the question, how long will be the duration of the occultation counted from the first external contact, the apparent radii of the satellites must be known. Owing to the irradiation they are greater at night than in daytime [1]) as several observers have actually found. The observations of the satellites of Jupiter being made nearly exclusively at night time, we will adopt the apparent radii holding for the night. I took the mean of the values found by See at the giant telescope at Washington on the one hand and that found by several observers on the other. (I have taken the values as summarised by See himself). For the reduction to the unit used throughout for these computations, *viz* the radius of the equator of Jupiter, this radius is taken = 18″·37 in accordance with Damoiseau.

	Diameter		Radius
I	1″·07 =	0ʳ·058	0ʳ·029
II	0 ·95	0 ·052	0 ·026
III	1 ·56	0 ·085	0 ·0425
IV	1 ·41	0 ·076	0 ·038

[1]) Vide e.g. T. J. J. See, Observations of the Diameters of the Satellites of Jupiter, and of Titan, the principal Satellite of Saturn, made with the 26 inch Refractor of the U. S. Naval Observatory, Washington; 19 Oct. 1901. *Astr. Nachrichten* N°. 3764, (21 Jan. 1902).

Therefore;

	Sum of the diameters	Sum of the radii
I + II . .	$0^r{\cdot}110$	$0^r{\cdot}055$
I + III . .	0·143	0·0715
I + IV . .	0·134	0·067
II + III . .	0·137	0·0685
II + IV . .	0·128	0·064
III + IV . .	0·161	0·0805

For the mean radii vectores we will take two figures more than did DAMOISEAU in his tables, and we will adopt for the purpose the values found by SOUILLART in DAMOISEAU's papers, (SOUILLART, second paper, Mémoires présentés par divers savants a l'Académie des Sciences, Tome XXX, 2me Série, 1889; p. 10) [1]).

I	6·0491,
II	9·6245,
III	15·3524,
IV	27·0027.

The result of our computation is, that the time between the first contact and the central occultation is:

for I and II	I and III	I and IV	II and III	II and IV	III and IV
$1^h{\cdot}324$,	$1^h{\cdot}245$,	$1^h{\cdot}103$,	$2^h{\cdot}263$,	$1^h{\cdot}774$,	$3^h{\cdot}725$;

between the central occultation and the second contact:

$1^h{\cdot}204$,	$1^h{\cdot}161$,	$1^h{\cdot}059$,	$2^h{\cdot}190$,	$1^h{\cdot}767$,	$3^h{\cdot}725$,

therefore in all

$2^h{\cdot}528$,	$2^h{\cdot}406$,	$2^h{\cdot}162$,	$4^h{\cdot}453$,	$3^h{\cdot}541$,	$7^h{\cdot}450$,

or

2^h32^m [2])	2^h24^m,	2^h10^m,	4^h27^m,	3^h32^m,	7^h27^m.

Still even these numbers do not represent the maximum of the time during which the two satellites may be seen as a single body. For we can imagine the case that the shortest distance becomes equal to $-(r + r')$, i. e. that between two central conjunctions there

[1]) According to SOUILLART, DAMOISEAU derived these numbers in the following way: He adopted the mean distance of IV, in accordance with POUND's determination = 496″·0, and took 18″·37 for Jupiter's semidiameter, so that, by division $r_{IV} = 27{\cdot}00102834$. The mean distances of the other satellites were then derived from the sidereal periods by the application of KEPPLER's third law. But to these mean distances he added the constant terms produced in the radii vectores by the perturbing force.

I beg leave to remark that 496″·0 : 18″·37 is not 27·00102834 but 27·000544366. Happily the 4th, 5th, 6th, 7th and 8th figure have no appreciable influence on our computations, nor probably on those of SOUILLART. For the rest the 2nd appendix, further below, may be consulted on such numbers of many decimals.

[2]) On June 4, 1908, such a conjunction must take place according to our computation. *Vide* the table further below.

occurs a contact on the other side. In this case the duration will, very nearly indeed, have to be multiplied by $\sqrt{2}$. It thus becomes for I and II, I and III, I and IV, II and III, II and IV, III and IV

$3^h{\cdot}574$, $3^h{\cdot}402$, $3^h{\cdot}057$, $6^h{\cdot}296$, $5^h{\cdot}006$, $10^h{\cdot}43$,

or:

3^h34^m, 3^h24^m, $3^h\,3^m$, 6^h18^m, $5^h\,0^m$, 10^h26^m.

These numbers hold only for those very rare occasions in which 1st. the occultation is central and 2nd. the rate of change of the elongation is equal or nearly so for the two satellites. As soon as there is some difference of latitude the time during which the two satellites are seen as a single body is of course smaller.

2nd. Appendix. *Investigation of the uncertainty, existing in the determination of the synodic periods of the satellites.*

In his introduction to the *Tables Ecliptiques*, p. XIX, DELAMBRE says: "Nous n'avons aucune observation d'éclipse antérieure à 1660". Now let us assume that the difference in time between the first eclipse observed in 1660 and the last observed in 1816, two years before the publication of these tables, (taking into account also the next ones in 1660 and the preceding ones in 1816) leaves an uncertainty, in the case of the four satellites, of 20, 30, 40 and 60 seconds, which will be too favourable rather than too unfavourable. If we divide this uncertainty by the number of synodic periods in 156 years, to wit 32193, 16032, 7951 and 3401, we get for the uncertainty of a single period

for I	for II	for III	for IV
$0^s{\cdot}00062$,	$0^s{\cdot}00188$,	$0^s{\cdot}0050$,	$0^s{\cdot}0176$.

Therefore, if we find that DELAMBRE gives these periods to 9 places of decimals of the second, we cannot attach much importance to the fact.

When DAMOISEAU, 20 years after DELAMBRE, published new eclipse-tables[1]) for the satellites of Jupiter, he adopted the period of I un-

[1]) The tables of DELAMBRE and DAMOISEAU were destined mainly to serve for the prediction, in the astronomical ephemerides, of the eclipses of the satellites caused by the shadow of Jupiter. It is for this reason that both he and DELAMBRE, united all those terms of the perturbations in longitude which have the same argument at the time of the opposition of the satellites, even though these arguments might be different for all other points in the orbit. Therefore it becomes necessary once more to separate these terms as soon as tables have to be computed from which may be derived the longitude and the radii vectores of the four satellites for any point of their orbits, tables such as have been given by BESSEL in his *Astronomische Untersuchungen* and by MARTH in the *Monthly Notices of the Royal Astronomical Society*, Vol. LI, (1891).

changed, but applied the following corrections to the remaining ones:

$$\text{II} + 0^{s}\cdot 005\ 127\ 374,$$
$$\text{III} + 0\cdot 029\ 084\ 25,$$
$$\text{IV} - 0\cdot 092\ 654\ 834,$$

the amount of which is even respectively nearly 3, nearly 6 and somewhat over 5 times that of the uncertainties derived just now. But even if we increase the number of intervening years from 156 to 176, our estimated uncertainties are only diminished by about $\frac{1}{9}$ of their amount. We thus conclude that these periods can only be considered to be determined with certainty:

that of I to 3 decimals of the second
„ „ II, III and IV to 2 decimals „ „ „ .

The Nautical Almanac, which, where it gives the superior conjunctions of the satellites, gives also the synodic periods, wisely confines itself to three decimals. The use of 9 decimals may therefore provisionally be taken for astronomical humbug. Some other instances of the same kind might be quoted *e. g.* the formerly well known constants, 20″·4451 for the aberration and 8″·57116 for the parallax of the sun!

3rd Appendix. *Meaning of the equations taken into account in the 2nd part of the tables of* Damoiseau.

On p. 321 we have referred to the 3rd appendix for information as to the equations which have been taken into account for each satellite in the second part of the tables of Damoiseau. We will now supply this information; we will denote by U, u_o, u_{I}, u_{II}, u_{III} and u_{IV} the mean longitudes of the sun, of Jupiter and of the four satellites; by π_o the longitude of the perihelium of Jupiter, by π' that of the Earth, by π_{III} and π_{IV} the perijovia of III and IV; by Π the longitude of the ascending node of Jupiter's equator on its orbit; finally by A_{II}, A_{III} and A_{IV} the longitudes of the ascending nodes of II, III and IV each on its own fixed plane.

In order to be able to supply the data following below we have taken the daily motion of the argument of each equation from the tables in the second part of Damoiseau. This amount was then multiplied by the synodic period expressed in days; the product thus obtained was then compared with the factor by which, in the first

part, p.p. (III), (V), (VII) and (VIII) the letter i (the number of synodic periods) is multiplied.

These daily motions are so nearly equal for several of the equations of II, III and IV that, in order to make them out, we must take from the tables the motions for a long interval, *e.g.* for 10 years, (duly taking into account the number of periods). These must then be divided by the number of days (10 years = 3652 or 3653 days). Multiplying this quotient by the synodic period in days, we get 360° + a fraction. The 360° are of no account; the fraction is the factor of i; we thus recognise which is the equation we have to deal with. In the preface of the second part of DAMOISEAU we look in vain for any information on the subject.

I. For this satellite five terms have been taken into account.

N°. 1 with an amplitude of 1°·16, is the equation of the velocity of light; its argument is $U - u_0$.

N°. 2, (amplitude 0°·29), is the equation caused by the ellipticity of Jupiter's orbit; the argument is the mean anomaly of Jupiter $u_0 - \pi_0$.

N°. 3 is 180° + the mean anomaly of the Earth, $U - \pi'$; by its aid and that of N°. 1 *i.e.* the difference in longitude between the Sun and Jupiter, we find, in the table of double entry IX, one term of the geocentric latitude of the satellite.

N°. 4 with an amplitude of 0°·45, shows the perturbation caused by II in the motion of I. The argument is $u_{\mathrm{I}} - u_{\mathrm{II}}$.

N°. 5, (amplitude 3°·07) gives the jovicentric latitude of I, necessary to find the second term of the geocentric latitude. The argument is $u_{\mathrm{I}} - \Delta_{\mathrm{I}}$.

II. Seven terms. N°. 1, 2 and 3 have the same arguments as the analogous terms for I; the amplitudes of N°. 1 and 2 are half those of I. The term of the latitude to be taken from IX, by the aid of 1 and 3, is of course the same for all the satellites.

N°. 4, (amplitude 1°·06), shows the perturbation caused by III in the motion of II. The argument is $u_{\mathrm{II}} - u_{\mathrm{III}}$.

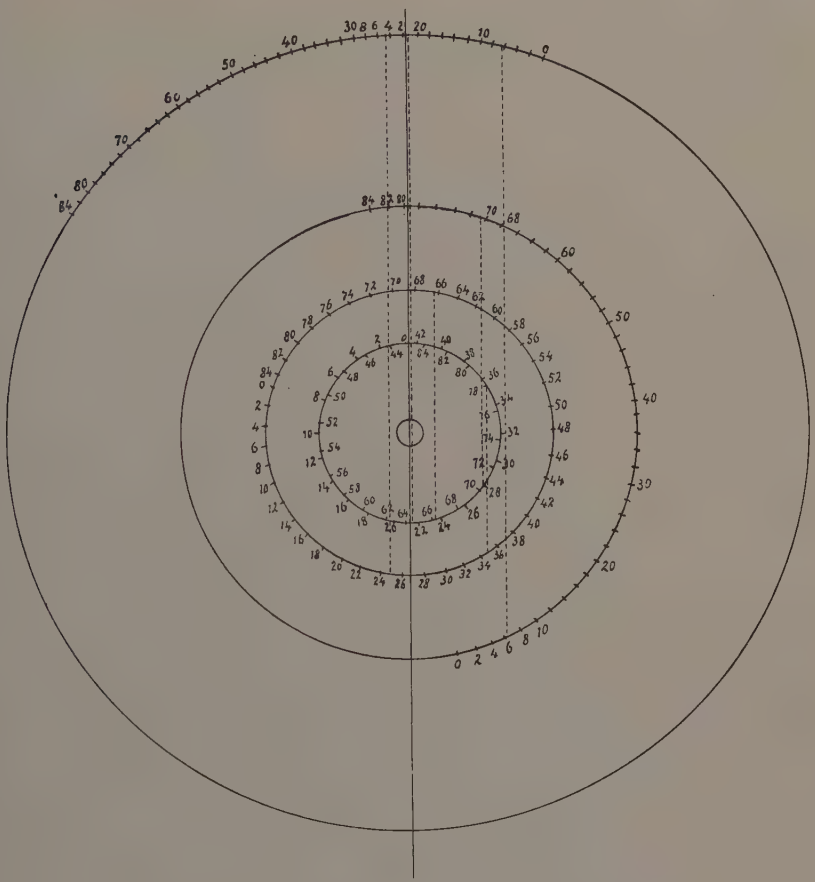
N°. 5, 6 and 7 serve for the latitude.

N°. 5, (amplitude 3°·05), has the argument $u_{\mathrm{II}} - \pi_{\mathrm{IV}}$;
N°. 6, („ 0 ·47), „ „ „ $u_{\mathrm{II}} - \Delta_{\mathrm{II}}$;
N°. 7. („ 0 ·03), „ „ „ $u_{\mathrm{II}} - \Delta_{\mathrm{III}}$.

III, Nine terms. Nos. 1, 2 and 3 are the same as for I and II; the amplitudes of N°. 1 and N°. 2 are 0°·29 and 0°·07.

J. A. C. OUDEMANS. „Mutual Occultations and Eclipses of the Satellites of Jupiter in 1908."

Starting point: the geocentric superior conjunction of I on July 12, 1908, at $11^h\ 2^m.3$ M. T. Greenwich $= 11^h\ 11^m.65$ M. T. Paris $= 23^h\ 11^m.65$ civil time Paris.



Scale $\frac{1}{30\,168\,000\,000}$; 1mm. = 8" heliocentric.

Unit: the radius of the earth's equator.

N°. 4, (amplitude 0°·07), has the same argument as N°. 4 for II, but it now shows the perturbation caused by II.

N°. 5, (amplitude 0°·15), is the equation of the centre; argument $u_{III}-\pi_{III}$.

N°. 6, (amplitude 0°·04), has the argument $u_{III}-\pi_{IV}$, it thus must account for a perturbation in III depending on the longitude of the perijovium of IV.

Nos. 7, 8 and 9, with the amplitudes 2°·98, 0°·18 and 0°·03, serve for the latitude. The arguments are respectively $u_{III}-\pi$, $u_{III}-\Lambda_{III}$ and $u_{III}-\Lambda_{IV}$.

IV. Seven terms.

Nos 1, 2 and 3 are similar to those of the preceding satellites.

N° 4, (amplitude 0°·83), is the equation of the centre, argument $u_{IV}-\pi_{IV}$.

Nos 5, 6 and 7 serve for the latitude. N° 5, (amplitude 2°·64) depends on the mean anomaly of Jupiter; its argument therefore is $u_0-\pi_0$.

N° 6, (amplitude 0°·24), depends on the argument of the latitude of the satellite itself; argument $u_{IV}-\Lambda_{IV}$.

N° 7, (amplitude 0°·04), is a minute perturbation, caused by III; its argument is $u_{IV}-\Lambda_{III}$.

Now in regard to the following table of the computed conjunctions.

The first column contains the ordinal numbers.

The second shows the epoch of the conjunction, accurate to the nearest minute, expressed in civil time of Paris. This time is reckoned from midnight and has been used by DAMOISEAU in his tables; it thus represents the direct result of our computations. In the cases that the computed time was just a certain number of minutes and a half, the half minute has been set down. By subtracting $12^u\,9^m$ or, where necessary, $12^u\,9^m\cdot35$, the mean time of Greenwich was found, which is contained in the third columm.

The 4th and the 5th columm contain the numbers of the occulted and the occulting satellite. The appended letters *f* and *n* show whether the satellite is *far* or *near* (vide supra p. 304). The satellite is *far* if its amplitude is between 9^s and 3^s, *near* if it is between 3^s and 9^s. Furthermore *ee* denotes an eastern elongation, for which the amplitude is about 3^s and *we* a western elongation, for which the amplitude differs little from 9^s.

At the conjunction the elongations, counted along the orbit of Jupiter, are equal; they are to be found in the next column. If the elongation is +, the satellite, as seen by a northern observer, using a terrestrial telescope, will be to the left of the planet. Therefore if he uses an inverting telescope, as is the rule for the observation of the heavenly bodies, he will see it to the right.

The three following columns contain the ordinates of the two satellites and their difference; northerly latitudes are positive. The tenth column shows the duration, which the eclipse would have, if the conjunction were central. In a few cases (Nos. 20, 23, 30, 48, 53 and 64), we find $y' = y$, consequently $y' - y = 0$. If the tables were correct these conjunctions would be central. But in testing the tables by the conjunctions observed by Messrs FAUTH, NIJLAND and STANLEY WILLIAMS the difference of the y's did not completely agree with the observations and even a small difference may considerably change the duration of any eventual occultation. Therefore, not to fill a column with figures, which, likely enough, may be contradicted by the observations, I omitted the value found by calculation for the true duration.

We remarked before (p. 308) that, if at all, any conjunction will be visible at a determined place of observation only for a short time, *viz.* between sunset and the setting of Jupiter. As a consequence the list will be of little use, unless observatories distributed over the whole of the earth cooperate in the work. The last column was added as a help to such cooperation. It contains on every line an observatory, at which the conjunction of that line will be visible. It is certainly desirable that other astronomers also, at observatories in the vicinity, examine whether the phenomenon will be visible, and, if so, prepare for its observation.

RESULTS.

Geocentric conjunctions of two satellites in June and July 1908.

No.	Civil time at Paris	Mean time at Greenwich	n = near, f = far: Occulted satellite	Occulting satellite	$x = x'$	Occulted satellite y	Occulting satellite y'	$y' - y$	Duration of central occultation	visible at
1	1 June 5h55m	31 May 17h46m	I_n	II_n	+2r18	−0r16	−0r12^5	+0,03^5	45m	Mt. Hamilton
2	1 » 18 25	1 June 6 16	I_{we}	II_n	−6,03	+0,31	+0,29	−0,02	11	Cape
3	2 » 1 55	1 » 13 46	I_f	IV_n	−3,15	+0,18	−0,12	−0,30	7	Washington
4	2 » 13 16	2 » 1 7	III_f	II_f	−8,51	+0,51	+0,38	−0,13	60	Madras, H. Kong
5	2 » 14 0	2 » 1 51	III_f	IV_n	−8,16	+0,51	+0,13	−0,38	11	Madras, H. Kong
6	2 » 14 4,5	2 » 1 55	II_f	IV_n	−8,20	+0,37	+0,13	−0,24	10	Madras, H. Kong
7	2 » 21 10	2 » 9 1	III_f	II_f	−4,54	+0,31	+0,20	−0,11	41	Utrecht
8	3 » 3 15	2 » 15 6	II_f	I_n	−0,33^5	0,00	−0,03	−0,03	4	At the same time ♂ ♆
9	3 » 3 50	2 » 15 41	III_f	I_n	−0,87	+0,15	0,00	−0,15	6	At the same time ♂ ♆
10	4 » 14 46	4 » 2 37	I_n	II_n	x=+5,01^5	−0,29	−0,25^5	+0,03^5	Two contacts at the same side. II just reaches central conjunction but then retraces its steps.	Madras
„	4 » 16 0	4 » 3 51	»	»	x'=+5,07 4,34					
„	4 » 17 15	4 » 5 6	»	»	x=+3,54 x'=+3,48^5	−0,23	−0,19^5	+0,03^5		
	The satellites will be visible as one body during nearly 2½ hours.									
11	5 » 7 51	4 » 19 42	I_{we}	II_f	−6,05	+0,33	+0,28	−0,05	12	Sydney
12	6 » 16 24	6 » 4 15	II_f	I_n	−0,48	0,00	−0,01	−0,01	4	At the same time ♂ ♆
13	6 » 18 11,5	6 » 6 2	II_f	III_n	+0,82	−0,06	−0,17	−0,11	6	and II eclipsed?
14	9 » 5 6	8 » 16 57	IV_f	II_f	−9,19	+0,59	+0,43	−0,16	13	Mt. Hamilton
15	10 » 4 26	9 » 16 17	IV_f	I_n	+0,29	+0,14	−0,05^5	−0,19^5	6	At the same time ♂ ♆
16	10 » 5 23	9 » 17 14	II_f	I_n	−0,58	+0,02	−0,01	−0,03	5	
17	10 » 6 28	9 » 18 18,5	III_f	I_n	−1,54	+0,15	+0,05	−0,10	6	Mt. Hamilton
18	11 » 3 0	10 » 14 51	IV_f	III_f	+9,01	−0,32	−0,52	−0,20	183	Washington
19	12 » 10 34	11 » 22 25	I_{we}	II_n	−6,03	+0,32	+0,28	−0,04	11	Wellington
20	13 » 18 46,5	13 » 6 37	II_f	I_n	−0,82	+0,01^5	+0,01^5	0,00	4	♂ ♆
21	13 » 21 37	13 » 9 28	II_f	III_n	+1,31	−0,07	−0,16	−0,09	6,5	II eclipsed (Utrecht)
22	15 » 23 55	15 » 11 46	I_{we}	II_n	−5,99	+0,32	+0,29	−0,03	10	(Atl. Ocean)
23	17 » 7 53	16 » 19 44	II_f	I_n	−0,92	+0,02	+0,02	0,00	4	♂ ♆

No.	Civil time at Paris	Mean time at Greenwich	n = near, f = far: Occulted satellite	Occulting satellite	$x = x'$	Occulted satellite y	Occulting satellite y'	$y' - y$	Duration of central occultation	Visible at
24	17 June 9h39m	16 June 21h30m	III_f	I_n	−2r44	+0r18	+0r10	−0r08	6m	Wellington
25	17 » 20 47	17 » 8 38	II_f	IV_n	+7,33	−0,37	−0,52	−0,15	9	Utrecht
26	18 » 0 33	17 » 12 24	III_f	IV_n	+5,75	−0,28	−0,44	−0,16	10	(Atl. Ocean)
27	18 » 7 30	17 » 19 21	I_f	IV_n	+2,86	−0,16	−0,30	−0,14	6,5	Sydney
28	18 » 8 28	17 » 20 18	III_f	II_{ee}	+9,52	−0,51	−0,45	+0,06	15	Sydney
29	19 » 13 13	19 » 1 4	I_{we}	II_n	−5,94	+0,32	+0,29	−0,03	9,5	Hong Kong
30	20 » 21 3	20 » 8 54	II_f	I_n	−1,07	+0,02	+0,04	+0,02	4	Utrecht
31	21 » 1 3	20 » 12 54	II_f	III_n	+1,79	−0,11	−0,15	−0,04	6,5	II eclipsed
32	21 » 11 54	20 » 23 45	I_f	III_n	−4,21	+0,24	+0,18	−0,06	7	Hong Kong
33	23 » 2 32	22 » 14 23	I_{we}	II_n	−5,90	+0,32	+0,29	−0,03	9	Washington
34	24 » 10 13	23 » 22 4	II_f	I_n	−1,22s	+0,03	+0,01	−0,02	4	Sydney
35	24 » 12 37	24 » 0 27	III_f	I_n	−3,21	+0,20	+0,16	−0,04	6,5	Hong Kong
36	26 » 11 17	25 » 23 8	IV_f	I_n	−5,60	+0,35	+0,30s	−0,04s	11	Hong Kong
37	26 » 13 43.5	26 » 1 34	IV_f	II_n	−4,61	+0,31	+0,25	−0,06	7	Madras, H. Kong
38	26 » 15 49	26 » 3 40	I_{we}	II_n	−5,84	+0,32	+0,30	−0,02	9	Madras
39	27 » 0 27	26 » 12 18	IV_f	I_f	−0,18	−0,01	+0,	+0,04	20	♂♃
40	27 » 13 44	27 » 1 35	IV_f	I_n	+5,66	−0,17	−0,3	−0,14	11	Madras, H. Kong
41	27 » 18 56	27 » 6 47	IV_f	III_n	+7,38	−0,20	−0,45	−0,25	10	Cape
42	27 » 23 24	27 » 11 14	II_f	I_n	−1,38	+0,03	+0,07	+0,04	4	(Atl. Ocean)
43	28 » 14 57	28 » 2 48	I_f	III_n	−3,49	+0,20	+0,17	−0,03	7	Madras
44	30 » 5 5.5	29 » 16 56	I_{we}	II_n	−5,75	+0,31	+0,31	0,00	4	Mt. Hamilton
45	1 July 12 34	1 July 0 25	II_f	I_n	−1,53	+0,09	+0,04	−0,05	4	Hong Kong
46	1 » 15 40	1 » 3 31	III_f	I_n	−3,93	+0,23	+0,21	−0,02	5	Central Asia
47	2 » 16 21.5	2 » 4 12	III_f	II_n	+9,14	−0,48	−0,42	+0,06	12	» »
48	3 » 18 21	3 » 6 12	I_f	II_n	−5,68	+0,31	+0,31	0,00	7	Cape
49	3 » 20 8	3 » 7 58.5	III_{ee}	IV_n	+15,30	−0,85	−0,79	+0,06	32	Berlin etc.
50	5 » 1 44	4 » 13 35.0	II_f	I_n	−1,67	+0,03	+0,10	+0,07	4	Washington
51	5 » 6 25	4 » 18 16	II_f	IV_n	+1,66	−0,12	−0,15	−0,03	7	II eclipsed

No.	Civil time at Paris	Mean time at Greenwich	n = near, f = far: Occulted satellite	n = near, f = far: Occulting satellite	$x = x'$	Occulted satellite y	Occulting satellite y'	$y' - y$	Duration of central occultation	visible at
52	5 July 8 h 2.5 m	4 July 19 h 53.2 m	II_f	III_n	$+2^r80$	-0^r17	-0^r29	-0^r12	7m	Sydney
53	5 » 17 58	5 » 5 49	I_n	III_n	−2,75	+0,15	+0,15	0,00	6	Cape
54	5 » 17 34	5 » 5 25	I_f	IV_n	−3,04	+0,14	+0,09	−0,05	7	Cape, Moscow
55	5 » 21 54	5 » 9 45	III_n	IV_n	−4,85	+0,28	$+0,16^5$	$-0,11^5$	79	Washington
56	7 » 7 37	6 » 19 28	I_f	II_n	−5,60	+0,30	+0,31	+0,01	8	Sydney
57	8 » 14 55	8 » 2 46	II_f	I_n	−1,82	+0,04	+0,11	+0,07	4	Madras
58	8 » 18 44.8	8 » 6 35.5	III_f	I_n	−4,64	$+0,26^5$	+0,25	$-0,01^5$	8	Moscow
59	9 » 20 13	9 » 8 4	III_f	II_n	+8,89	−0,50	−0,40	+0,10	11,5	Berlin, etc.
60	10 » 20 52	10 » 8 47	I_f	II_n	−5,53	+0,31	+0,30	−0,01	7	Utrecht
61	12 » 4 6.5	11 » 15 57	II_f	I_n	$-1,97^5$	+0,04	+0,13	+0,09	4	Mt Hamilton
62	12 » 20 57	12 » 8 47	I_f	III_n	−1,95	+0,11	$+0,13^7$	$+0,02^5$	6	Utrecht
63	13 » 5 25	12 » 17 15	IV_f	III_n	−6,50	+0,35	+0,40	+0,05	10,5	Pacific
64	13 » 20 39	13 » 8 30	IV_f	I_n	−0,27	+0,05	+0,05	0,00	6	♂☌
65	14 » 0 12.5	13 » 12 3	IV_f	II_n	$+1,20^5$	$-0,01^5$	$+0,01^5$	+0,03	4	IV eclipsed
66	14 » 10 8.5	13 » 21 59	I_f	II_n	−5,42	+0,29	+0,32	+0,03	7	Perth
67	15 » 17 17	15 » 5 7	II_f	I_n	−2,12	+0,04	+0,14	+0,10	4	Cape
68	15 » 22 1	15 » 9 51	III_f	I_n	−5,27	+0,28	+0,30	+0,02	9	Berlin, etc.
69	17 » 23 11	17 » 11 2	I_f	II_n	−5,34	+0,29	$+0,31^5$	$+0,02^5$	6,5	Charkow
70	19 » 6 30	18 » 18 21	II_f	I_n	−2,22	+0,06	+0,14	+0,08	4	Pacific
71	19 » 15 5	19 » 2 56	II_f	III_n	+3,77	−0,22	−0,15	+0,07	7	Central Asia
72	19 » 23 56	19 » 11 47	I_f	III_n	$-1,13^5$	+0,05	+0,12	+0,07	6	Astrachan

Occultation of II_f by I_r, on July 15, 1908 at 17^h27^m civil time Paris.

	I						II				
	M. Longitᵉ	1	2	3	4	5	M. Longitᵉ	4	5	6	7
1858	9ˢ 5°74	7ˢ27°5	1ˢ 0°8	0ˢ 1°4	0ˢ18°1	10ˢ20°9	8ˢ17°66	3ˢ 9°0	10ˢ 2°8	0ˢ18°6	11ˢ28°5
1880	6 9,80	9 19,2	11 8,4	0 0,8	2 5,9	7 24,6	4 3,87	10 2,9	5 18,7	5 0,2	9 10,6
—1830	0 6,20	11 22,6	3 8,9	11 28,6	10 4,2	10 20,7	2 2,03	2 2,1	0 16,5	9 8,9	1 2,3
July	3 21,51	5 13,4	0 15,0	5 28,4	4 2,7	3 21.5	11 18,83	8 1,3	11 18,8	11 24,8	11 20,1
July 1908	7 13,25	10 22,7	4 3 1	5 29,2	5 0,9	8 27,7	2 12,39	11 15.3	3 26,8	2 22,5	10 1,5
15d	10 28,85	0 12,6	0 1,2	0 13,8	11 19,6	10 28,8	11 9,25	11 24,8	11 9,2	11 9,7	11 0,3
17h	4 24,14	0,6		0,7	2 12,3	4 24,1	1 11,81	1 6,2	2 11,8	2 11,8	2 11,8
27m	3,82				1,9	3,8	1,90	0,9	1,9	1,9	1,9
	11 10,06	11 5,9	4 4,3	6 13,7	7 4,7	0 24,4	4 5,35	0 17,2	5 19,7	4 15,9	11 24,5
1	— 1,06					1	— 0,52				
2	— 0,16					2	— 0,08		5 + 0°53		
4	+ 0,43					4	+ 0,60		6 — 0,32		
	11 9,27			5 + 1°27			4 5,35		7 0,00		
S—G	4 17,37			— IX + 0,05		S—G	4 17,37		— IX + 0,05		
Amplitude	6 21,90			+ 1,32		Amplitude	11 17,98		+ 0,26		
			$x = -2^r,26$	$y = +0^r,14$					$x' = -2^r 00$	$y' = +0^r,04$	

$$-2^r,26 - 0,830\,\Delta t = -2^r,00 + 0,691\,\Delta t$$
$$-1,521\,\Delta t = +0\,,26$$
$$\Delta t = -0^h,172 = -10^m,3$$
$$x = -2^r,12.$$

Result: 15 July at $17^h16^m{\cdot}7$ Civil time at Paris,
$= 5\ 16\ {\cdot}7$ Mean „ „ „ „
$= 5\ 7\ {\cdot}35$ „ „ „ Greenwich.

Due account has been taken of the fact that the number of bissextile days between 1858 and 1908 differs from that between 1830 and 1880. For between (1 January) 1830 and (1 January) 1880 we have 12 bissextile days, against 11 between (1 January) 1858 and (1 January) 1908. The difference has been allowed for by treating 1908 as if it were a *common* year in the line of *July*, which contains the motions for the months January to June (inclusive). For on the pages 201, 209, 217 and 225, which contain the motion of the four satellites for the several months, we find two tables, one headed "Années communes", the other headed "Années bissextiles".

The fifth line of this computation (July 1908) may be used for all the conjunctions of this month.

As a second example we will take the occultation of III_f by II_f, for which our drawing gave June 2, 1908, $12^h9^m{\cdot}3$ civil time at Paris as a first approximation.

	II								III					
	Mean Longitᵉ	1	2	3	4	5	6	7	Mean Longitᵉ	5	6	7	8	9
1908 June	$9^s\ 1°15$	$9^s\ 25°6$	$4^s\ 0°6$	$4^s\ 29°6$	$8^s\ 13°6$	$10^s\ 15°6$	$9^s\ 10°3$	$4^s\ 20°0$	$0^s\ 17°45$	$0^s\ 7°7$	$2^s\ 24°3$	$2^s\ 2°0$	$8^s\ 6°3$	$6^s\ 26°4$
2^d	3 11,37	0 0,9	0 0,1	0 1,0	1 21,1	3 11,4	3 11,4	3 11,4	1 20,32	1 20,3	1 20,3	1 20,3	1 20,3	1 20,3
12^h	1 20,69	0,5		0,5	0 25,5	1 20,7	1 20,7	1 20,7	0 25,16	25,2	25,2	25,2	25,2	25,2
$9^m{,}3$	0,63				0,3	0,6	0,6	0,6	0,32	0,3	0,3	0 3	0,3	0,3
	2 3,84	9 27,0	4 0,7	5 1,1	11 0,5	3 18,3	2 13,0	9 22,7	3 3,25	2 23,5	5 10,1	4 17,8	10 22,1	9 12,2
1	− 0,28							1	− 0,13					
2	− 0,07							2	− 0,04					
4	− 0,91				5	+ 2°89		4	+ 0,04					
					6	− 0,45		5	+ 0,15					
	2 2,58				7	+ 0,03		6	+ 0,01			7	+ 2°01	
S—G	4 9,25				− IX	− 0,11						8	+ 0,11	
						+ 2,36			3 3,28			9	+ 0,03	
Amplitude	9 23,33							S—G	4 9,25			− IX	− 0,11	
				$x = -8^r{,}84$		$y = +0^r{,}40$								
								Amplitude	10 24,03				+ 2,04	
											$x' = -9^r{,}01$		$y' = +0^r{,}055$	

$$-8{,}84 + 0{,}280\,\Delta t = -9{,}01 + 0{,}453\,\Delta t$$
$$+0{,}17 = +0{,}173\,\Delta t$$
$$\Delta t = +0^m{\cdot}983 = +59^m$$

First approximation : 12 9 ,3,

Second approximation : 13 8 ,3.

If we repeat the computation with this value, we still do not get equality of the elongations. The amplitude of II becomes $9^s27^m{\cdot}56$, that for III $10^s26^m{\cdot}08$. Furthermore we find :

$$-8^r{,}535 + 0^r{,}327\,\Delta t = -8^r{,}56 + 0^r{,}4645\,\Delta t$$
$$+0{,}025 = +0{,}1365\,\Delta t$$
$$\Delta t = +0^h{,}182 = +10^m{,}9$$

13 8 ,3

Third approximation : $13^h\ 19^m{\cdot}2$ Civil time at Paris,
$= 1\ 19\cdot2$ Mean „ „ „ ,
$= 1\ 9\cdot85$ „ „ „ Greenwich.

Pathology. — *"On the Amboceptors of an Anti-streptococcus serum."* By H. EYSBROEK. (From the Pathological Institute of Utrecht). (Communicated by Prof. C. H. H. SPRONCK.)

(Communicated in the meeting of September 29, 1906).

As is known, there exists in the serum of an animal which is treated with the bloodcorpuscles of an animal of another species, a substance, which is capable of bringing the bloodcorpuscles of the second animal to solution with the aid of another substance, which is already present in normal serum. The first substance, which only appears in immune-sera, is thermostatic and is named differently by different investigators, according to the idea which they make of its influence (Amboceptor of EHRLICH, Substance sensibilisatrice of BORDET, Fixateur of METCHNIKOFF). The other substance, which normally is present in all sorts of sera in greater or smaller quantities, is easily made inactive by heating to 55—56° C. or by being exposed to light. It has been proved, that the last mentioned substance is identical with a bactericidal substance, demonstrated by FODOR [1]) and FLÜGGE [2]) in normal blood-serum, to which is given the name of alexin by BUCHNER. Next to this name at present the denominations complement (EHRLICH) and cytase (METCHNIKOFF) are used.

Had METCHNIKOFF in 1889 already pointed out the analogy between hemolytic and bacteriolytic processes, later investigations have completely comfirmed this supposition.

In 1901 BORDET and GENGOU [3]) published a method to demonstrate the presence of a "substance sensibilisatrice" in the serum of an animal, which was immunized against a certain micro-organism, by means of a combination with the complement. At the same time they found, that this amboceptor is specific; for instance, the amboceptor, present in the bloodserum of animals which were immunized against cholera spirilla, is indeed active against the cholera spirilla, but not against other bacteria, such as the typhoid bacilli.

On the other hand one is capable of distinguishing with the aid of an amboceptor at hand, the micro-organism belonging to it from others, by means of a combination with the complement.

Using the above mentioned method of BORDET—GENGOU, BESREDKA [4]) succeeded in pointing out an amboceptor also in an anti-streptococcus

[1]) Deutsche Med. Wochenschrift, 1887, N°. 34, S. 745.
[2]) Zeitschrift für Hygiene, Bd. IV, S. 208.
[3]) Annales de l'Inst. Pasteur, T. 15, 1901, p. 289.
[4]) Annales de l'Inst. Pasteur, T. 18, 1904, p. 363.

serum prepared by himself. This serum was obtained from a horse, which for some time was injected intravenously with a mixture of 6—8 different streptococci, which but for one exception originated immediately, so, without passage through animals, from pathological processes of man. Besides, he has made use of the presence of an amboceptor in his serum to investigate, whether it might be possible to separate different races of streptococci from each other with the aid of this substance.

Among the principal difficulties, which are still experienced in the preparation of an anti-streptococcus serum, must be mentioned in the first place, that the streptococci proceeding directly from pathological processes of man and being very virulent for him (scarlatina, erysipelas, septicemia etc.) possess in general for our common test-animals a comparatively small degree of virulency. By this the preparation of a very powerful serum is somewhat impeded and on the other hand it is almost impossible to controll the obtained serum. In the second place the question prevails, whether all streptococci, cultivated from different processes of disease, must be regarded as representatives of one and the same species, and to be taken as varieties, or that the mutual affinity is much smaller. A solution of this question in such a sense, that it might be possible to come to a rational sub-division in the large group of the pathogenic streptococci, would be of great importance for the bloodserum-therapy.

Some years ago SCHOTTMÜLLER [1]) tried to give a new division, based on biological grounds instead of the older morphological division in streptococcus longus and streptococcus brevis (VON LINGELSHEIM [2]), BEHRING [3]). By cultivating different races of streptococci on blood-agar, he was enabled to discern two types: firstly dark grey colonies with lucid area, secondly greenish ones without area. The streptococci, belonging to the first group, are very virulent for man and are found in erysipelas, septicemia, scarlatina, phlegmon etc., while those, belonging to the second group, are generally less pathogenic for man and animals. Therefore SCHOTTMÜLLER divides the pathogenic streptococci as follows:

1. Streptococcus pyogenes s. erysipelatos.
2. Streptococcus mitior s. viridans.
3. Streptococcus mucosus.

Several other investigators (EUG. FRAENKEL [4]), SILBERSTROM [5]),

[1]) Münch. Med. Wochenschrift, 1903, N°. 20, S. 849; N°. 21, S. 909.
[2]) Zeitschrift für Hygiene, Bd. X, S. 331.
[3]) Centralblatt. für Bakteriologie, Bd. 12, S. 192.
[4]) Münch. Med. Wochenschrift, 1905, N°. 12, S. 548; N°. 39, S. 1869.
[5]) Centralblatt für Bakt., 1e Abth., Orig., Bd. 41, S. 409.

BAUMANN [1]) have latterly come to the same result in an almost similar way.

BESREDKA [2]) on the contrary tried to separate the different streptococci from each other with the aid of the method of the combination with the complement. The conclusion to which he comes, is, that the "substances sensibilisatrices" present in his serum, are "rigoureusement" specific; that the serum of a horse, immunized with the streptococcus *A*, only contains the amboceptor *A''*, which corresponds with that special streptococcus. Thus he found this amboceptor *A''* not only active against the streptococcus *A*, but also against other races (*B*, *C*), from which BESREDKA decides on the identity or at least on the near relationship of the above mentioned streptococci *A*, *B* and *C*.

According to these results, some experiments have been taken by me, to trace, in how far a separation of the different pathogenic streptococci is really possible by means of the specific action of the amboceptors.

The anti-streptococcus serum, which I used, Prof. SPRONCK willingly provided me with, for which I offer him my best thanks as well as for his further assistance in my work.

The above mentioned serum originated from a horse, which was injected for a great length of time viz. from Jan. 1905 till July 1906, with a number of specimens of streptococci and staphylococci of different origin. These injections, which were subcutaneous, took place weekly. The quantities used were gradually increased during the first months; whilst after that on an average 40—60 c.C. of a mixture, composed of even parts of a culture in ascitic-bouillon of the different streptococci and of a bouillon-culture of the staphylococci, were administered. The mixture was twice heated for half an hour to 55° C. Strepto- as well as staphylococci originated directly from man, without passage through animals.

That the serum really possesses curative qualities is evident, not only from observations in the clinical surgery, but also from experiments upon animals. Rabbits, which were injected with a mixture of strepto- and staphylococci, could be kept alive by administering comparatively small quantities of the anti-streptococcus serum, whilst animals used for controll died shortly after.

The method, followed by me, is that of BORDET—GENGOU [3]). For

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[1]) Münch. Med. Wochenschrift, 1906, N°. 24, S. 1193.

[2]) l. c.

[3]) l. c.

each experiment six tubes were used, which contained consecutively [1]):

N°. 1: $^2/_{10}$ c.C. complement, $^1/_2$ c.C. emulsion of streptococci, $^1/_2$ c.C. anti-streptococcus serum.

N°. 2: $^2/_{10}$ c.C. compl., $^1/_2$ c.C. emulsion of str., $^1/_2$ c.C. normal horse-serum.

N°. 3: $^2/_{10}$ c.C. compl., $^1/_2$ c.C. physiological NaCl, $^1/_2$ c.C. anti-streptococcus serum.

N°. 4: $^2/_{10}$ c.C. compl., $^1/_2$ c.C. physiol. NaCl, $^1/_2$ c.C. normal horse-serum.

N°. 5: $^2/_{10}$ c.C. physiol. NaCl, $^1/_2$ c.C. emulsion of str., $^1/_2$ c.C. anti-streptococcus serum.

N°. 6: $^2/_{10}$ c.C. physiol. NaCl, $^1/_2$ c.C. emulsion of str., $^1/_2$ c.C. normal horse-serum.

The tubes are stirred and then remain at the same temperature as the room. Afler 3—5 hours to each of the tubes is added $^2/_{10}$ c.C. of a mixture, composed of 2 c.C. of hemolytic serum and 1 c.C. corpuscles of a rabbit, which were suspended in physiol. NaCl to remove the adherent serum. Very soon, mostly within ten minutes the tubes 2, 3 and 4 distinctly show the phenomenon of hemolyse; which is naturally not brought about in tubes 5 and 6, the complement being absent. The absence or presence of an amboceptor in the examined serum is proved by the existence or non-existence of the hemolyse in the first tube.

It is necessary to repeat all these controll-experiments each time; firstly, because some streptococci produce a hemolysin at their growth; secondly, because bacteria are able to combine the complement without the aid of an amboceptor, although in a much smaller degree. This may be observed very distinctly in vitro; for instance: in six tubes successive dilutions of a culture of diphtheria bacilli were made; to each tube $^2/_{10}$ c.C. of the complement was added. After three hours $^2/_{10}$ c.C. of a mixture, composed of 2 c.C. of hemolytic serum (heated to 56° C.) and 1 c.C. corpuscles of a rabbit, suspended in physiol. NaCl, was added. The result after half an hour was as

[1]) As complement, the fresh blood-serum of a guinea-pig was used. The streptococci, which were to be examined, were cultivated on LOEFFLER's coagulated blood-serum and after 24 hours suspended in physiological NaCl to a homogeneous emulsion. The antistreptococcus serum was heated in advance for one hour to 56° C., as well as the fresh normal horse-serum, used for controll, and the hemolytic serum originating from guinea-pigs, which were treated 3 or 4 times with 5 c.C. of defibrinated blood of rabbits. The physiological NaCl used, was always a solution f 0,9%.

follows: no hemolyse in the first (least diluted) tube, a little hemolyse in the 2nd tube, more and more hemolyse in tubes 3, 4 and 5 whilst in the sixth (most diluted) tube it was perfect.

The same experiment was made with different other bacteria with a similar result.

It may be easily understood, that it sometimes occurs, that no hemolyse is formed in the first of the tubes, used in the method of BORDET—GENGOU, in consequence of a surplus of bacteria, as is seen by the absence of the hemolyse in the second tube at the same time. Without the controll-tubes, one might wrongly decide on the presence of an amboceptor in the examined serum.

In the first place an investigation was made, whether in the anti-streptococcus-serum, used by me, an amboceptor was present against some five streptococci used at the immunization. The result was positive. After this, different other streptococci were investigated. These streptococci originated directly from different diseases of man, such as: scarlatine, cholecystitis, septicemia, febris puerperalis, angina, and had not served at the immunization. Among these streptococci there were some of patients who during their lifetime had been injected with the same anti-streptococcus serum, but without success.

The latter streptococci were cultivated from the blood or from the spleen post mortem. Others were cultivated from patients with whom the injections of the serum had had a very distinct curative effect. It was therefore supposed that against the first streptococci no amboceptor would be found in the anti-streptococcus serum.

The investigation however did not confirm this supposition. *All streptococci, no matter what their origin, showed a strong combination with the complement under the influence of the anti-streptococcus serum.*

Keeping to the specific of the amboceptors, the conclusion of BESREDKA [1]) might be accepted, regarding all the latter streptococci as identic or at least closely related to those used at the immunization. Continued experiments with some pathogenic streptococci originating from animals, have led to a different interpretation. A streptococcus was used, which was cultivated from the lungs of a guinea-pig, which had died spontaneously from pneumonia; further the well-known streptococcus equi and a couple of other streptococci, which were cultivated with the Str. equi from pus, originating from horses suffering from strangles. Also against the latter streptococci, the presence of an amboceptor in the used serum was an undoubted fact.

Considering that the str. equi by its qualities, apparently from its

[1]) l. c.

deviating growth on the usual culture-media, shows very distinct differences from the other pathogenic streptococci, whether from man or from animals, the conclusion is at hand, that at least in the anti-streptococcus serum, used by me, very little of the specific working of the amboceptor is left. It is however quite possible that all pathogenic streptococci, originating from man as well as from animals, are very closely related, by which supposition one might keep at least to the specific of the amboceptors.

However later experiments have shown that the anti-streptococcus serum is also active against micro-organisms, which do not belong to the streptococci viz, pneumococci and meningococci.

By the above is fully shown, that the specific action of the amboceptors in the serum of a horse to which large quantities of streptococci have been administered for a very long time, has strongly decreased and made room for a more general action. Probably this general working might be put to the account of one and the same amboceptor, although the presence of more amboceptors in the same cannot be denied.

The above mentioned serum exercises, though in a small degree, also a distinctly sensitive action on anthrax, typhoid and tubercle bacilli.

From the above it appears, that the method of the combination with the complement of BORDET—GENGOU, is not to be used, if it is necessary to distinguish nearly related bacteria from each other, which in other ways are also difficult to separate.

Granted that it must be accepted, that such a diminution of the specific activity only takes place with sera of animals which have been treated for a great length of time, so that the specific activity of the amboceptor is more asserted in proportion to the shorter time in which the animals are immunized, it is evident here, that there is no question about a certain method being used, because one never knows, — and this is also the case with sera of animals which have only shortly been immunized — how far the specific action extends. Even if it may be accepted that the horse, from whom the anti-streptococcus serum originates, is a most favourable test-animal as regards the forming of anti-bodies, then the above mentioned facts would remain the same.

DOPTER[1]) has recently found, that the amboceptor, present in the serum of a horse which has been treated with dysenteria bacilli (type SHIGA) during 18 months, next to the action on these bacilli,

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[1]) Annales de l'Inst. Pasteur, T. 19. 1905, p. 753.

also presented the self-same effect against the so-called pseudo- or para-dysenteria bacilli (type FLEXNER, KRUSE). Asserting the specific activity of the amboceptor, he decides on "l'unité specifique" of the dysenteria bacilli. This conclusion appears to me, looking at the above, very venturesome.

At the same time it is evident, that we must not attach too much importance to the presence of an amboceptor in a serum for the effect of that serum. It is not to be accepted, that the anti-streptococcus serum will have a favourable effect on patients suffering from pneumonia, typhus, anthrax etc. although a certain effect is to be observed in vitro against the respective causes of these diseases. I purposely treated this for anthrax bacilli. Different guinea-pigs of nearly the same weight received partly a small quantity of anti-streptococcus serum (2—3 c.C.), which contained some anthrax bacilli (one eye of a deluted twelve hours, old culture on bouillon-agar), partly normal horse-serum (2—3 c.C.) with an equal dose of anthrax bacilli. A favourable effect of the anti-streptococcus serum compared to normal serum was never perceptible. The animals died generally about the same time, within 48 hours.

Yet PREDTETSCHENSKY [1]), who has made such investigations with rabbits, is of opinion that a favourable effect can be perceived from anti-diphtheria as well as from anti-streptococcus serum, but the colossal quantities of serum, which he used, justify the supposition, that here is only question of the favourable effect, which, as is known, is already produced in several cases by the injection of normal horse-serum.

It is therefore not permissible, to ascribe a favourable effect to a serum by force of the presence of an amboceptor, still less, to base on this a quantitative method for the determination of the force of such a serum, such as KOLLE and WASSERMANN[2]) do with regard to a meningococcus serum prepared by them. In the meningococcus serum of JOCHMANN (E. MERCK) the presence of an amboceptor could not only be clearly discerned against meningococci, but also, naturally in a smaller degree, against some streptococci.

The question, if such a diminishing of the specific activity in relation to a prolonged administering of antigens is known for other substances in immune-sera too, must be answered in the affirmative. This is especially the case with regard to the precipitins. It is well known that it is not possible to obtain them absolutely specific. Thus NUTTALL [3]) was able to get a precipitation with the blood-serum of all kinds

[1]) Centralblatt für Bakt., 1e Abth., Ref., Bd. 38, S. 395.

[2]) Deutsche Med. Wochenschrift, 1906, n° 16, S. 609.

[3]) Blood immunity and blood relationship, Cambridge, 1904, p. 74, 135, 409.

of mammals even with a very strong precipitin-serum, which was obtained with and against an arbitrary mammifer-albumen ("mammalian reaction"). HAUSER [1]) comes to a similar result; only quantitative differences remain.

Also with relation to the amboceptor such a diminution of the specific action seems to me sufficiently well pointed out.

Physics. — "*Arbitrary distribution of light in dispersion bands, and its bearing on spectroscopy and astrophysics.*" By Prof. W. H. JULIUS.

In experimental spectroscopy as well as in the application of its results to astrophysical problems, it is customary to draw conclusions from the appearance and behaviour of spectral lines, as to the temperature, density and motion of gases in or near the source of light.

These conclusions must in many cases be entirely wrong, if the origin of the dark lines is exclusively sought in absorption and that of the bright ones exclusively in selective emission, without taking into account the fact that the distribution of light in the spectrum is also dependent on the anomalous dispersion of the rays in the absorbing medium.

It is not in exceptional cases only that this influence makes itself felt. Of the vapours of many metals it is already known that they bring about anomalous dispersion with those kinds of light that belong to the neighbourhood of several of their absorption lines [2]). In all these cases the appearance of the absorption lines must to a greater or less extent be modified by the above mentioned influence, since the mass of vapour, traversed by the light, is never quite homogeneous.

Hence it is necessary, separately to investigate the effect of dispersion on spectral lines; we must try to separate it entirely from the phenomena of pure emission and absorption.

A first attempt in this direction were the formerly described experiments with a long sodium flame [3]), in which a beam of white

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[1]) Münch. Med. Wochenschrift, 1904, n° 7, S. 289.

[2]) After WOOD, LUMMER and PRINGSHEIM, EBERT, especially PUCCIANTI has investigated the anomalous dispersion of various metallic vapours. In Nuovo Cimento. Serie V, Vol. IX, p. 303 (1905) PUCCIANTI describes over a hundred lines, showing the phenomenon.

[3]) W. H. JULIUS, "Dispersion bands in absorption spectra." Proc. Roy. Acad. Amst. VII, p. 134—140 (1904).

light alternately travelled along different paths through that flame. With these relative displacements of beam and flame the rays of the anomalously dispersed light were much more bent, on account of the uneven distribution of the sodium vapour, than the other rays of the spectrum; absorption and emission changed relatively little. The result was, that the distribution of the light in the neighbourhood of D_1 and D_2 could be made very strongly asymmetrical, which could easily be explained in all details as the result of curvature of the rays. The existence of "dispersion bands" was thus proved beyond doubt.

But the pure effect of emission and absorption was not absolutely constant in these experiments and concerning the density of the sodium vapour in the different parts of the flame only conjectures could be made. Moreover, the whirling ascent of the hot gases caused all rays, also those which suffered no anomalous dispersion, sensibly to deviate from the straight line, so that the phenomena were too complicate and variable to show the effect of dispersion strictly separated from that of emission and absorption.

So our object was to obtain a mass of vapour as homogeneous as possible and, besides, an arrangement that would allow us to bring about arbitrarily, in this vapour, local differences of density in such a manner, that the average density was not materially altered. The absorbing power might then be regarded as constant. At the same time it would be desirable to investigate the vapour at a relatively low temperature, so that its emission spectrum had not to be reckoned with.

In a series of fine investigations on the refractive power and the fluorescence of sodium vapour R. W. Wood[1]) caused the vapour to be developed in an electrically heated vacuum tube. It appeared possible, by adjusting the current, to keep the density of the vapour very constant. Availing myself of this experience I made the following arrangement for the investigation of dispersion bands.

Apparatus.

NN' (see fig. 1) is a nickel tube of 60 centimetres length, 5.5 cms. diameter and 0,07 cm. thickness. Its middle part, having a length of 30 cms., is placed inside an electrical furnace of Heraeus (pattern E 3). Over its extremities covers are placed, the edges of which fit into circular rims, soldered to the tube, and which consequently

[1]) R. W. Wood, Phil. Mag. [6], 3, p. 128; 6, p. 362.

shut air-tight when the rims are filled with cement. When the furnace is in action a steady current of water, passing through the two mantles M and M', keeps the ends of the tube cool. Each of the two caps has a rectangular plate glass window and also, on both sides of this, openings a and b (b' and a'), placed diametrically

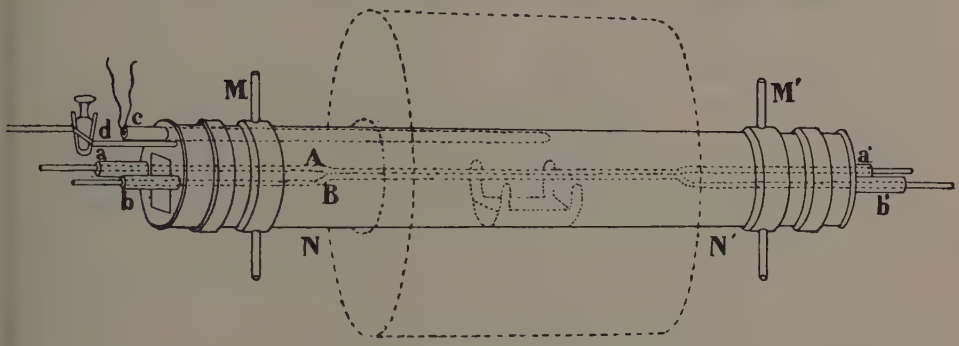

Fig. 1.

opposite to each other and provided with short brass tubes, the purpose of which will appear presently. Moreover in one of the two caps (see also fig, 2) two other short tubes c and d are fastened in openings: through c the porcelain tube of a LE CHATELIER pyrometer is fitted air-tight, while on d a glass cock with mercury lock is cemented, leading to a manometer and a Geryk air-pump. As soon as the sodium (a carefully cleaned piece of about 7 grammes) had been pushed to the middle of the tube in a small nickel dish provided with elastic rings, the tube had been immediately closed and exhausted.

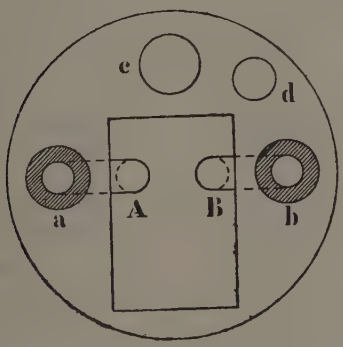

Fig. 2.

We shall now describe the arrangement by which inside the mass of vapour arbitrary inequalities in the density distribution were produced. It consists of two nickel tubes A and B of 0,5 cm. diameter, leading from a to a' and from b to b' and so bent that in the heated middle part of the wide tube they run parallel over a length of 30 centimetres at a distance of only 0.8 cms. In the four openings of the caps, A and B are fastened air-tight by means of rubber packing, This kind of connection leaves some play so that by temperature differences between the wide and the narrow tubes these latter need

not alter their shape through tension. At the same time the rubber insulates A and B electrically from NN'. The four ends of the narrow tubes which stick out are kept cool by mantles with streaming water (these are not represented in the figure).

If now an electric current is passed through A or B, the temperature of this tube rises a little above that of its surroundings; if an air-current is passed through it, the temperature falls a little below that of its surroundings. The intensities of the currents and, consequently, the differences of temperature can in either case be easily regulated and kept constant for a long time.

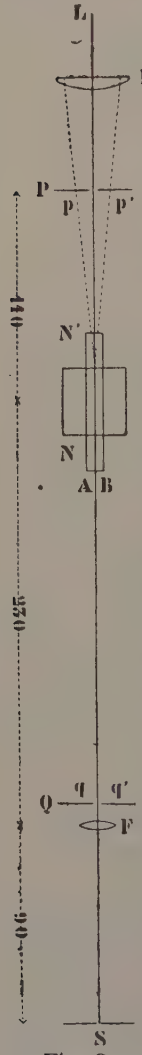

Fig. 3.

Fig 3 gives a sketch of the whole arrangement. The light of the positive carbon L is concentrated by the lens E on a screen Q, having a slit-shaped aperture of adjustable breadth. The lens F forms in the plane of the slit S of the spectrograph a sharp image of the diaphragm P. The optical axis of the two lenses passes through the middle of the tube containing the sodium vapour, exactly between the two small tubes A and B.

If now the opening in the diaphragm P has the shape of a vertical narrow slit and if its image falls exactly on the slit of the spectograph, then in this latter the continuous spectrum of the arc-light appears with great brightness. If the tube NN' is not heated, D_1 and D_2 are seen as extremely fine dark lines, attributed to absorption by the sodium, which is always present in the neighbourhood of the carbons. In order that this phenomenon might always be present in the field of view of the spectograph as a comparison spectrum, also when the tube is heated, a small totally reflecting prism was placed before part of the slit S, to which part of the principal beam of light was led by a simple combination of lenses and mirrors without passing the electric furnace. So on each photograph that was taken the unmodified spectrum of the source is also seen.

The spectral arrangement used consists of a plane diffraction grating 10 cms. diameter (ruled surface 8 by 5 cms.) with 14436 lines to the inch, and two silvered mirrors of Zeiss; the collimator mirror has a focal distance of 150 cms., the other of 250 cms. Most of the work was done in the second spectrum.

When heating the sodium for the first time a pretty large quantity of gas escaped from it (according to WOOD hydrogen), which of course was pumped off. After the apparatus had functionated a couple of times, the tension within the tube remained for weeks less than 1 mm. of mercury, also during the heating, which, in the experiments described in this paper, never went beyond 450°. The inner wall of NN' and also the small tubes A and B are after a short time covered with a layer of condensed sodium, which favours the homogeneous development of the vapour in subsequent heatings. It is remarkable that scarcely any sodium condenses on the parts of the tube that stick out of the furnace, so that also the windows remain perfectly clear. The density of saturated sodium vapour at temperatures between 368° and 420° has been experimentally determined by JEWETT [1]). He gives the following table.

temperature	density
368°	0.00000009
373	0.00000020
376	0.00000035
380	0.00000043
385	0.00000103
387	0.00000135
390	0.00000160
395	0.00000270
400	0.00000350
406	0.00000480
408	0.00000543
412	0.00000590
418	0.00000714
420	0.00000750

These densities are of the same order of magnitude as those of mercury vapour between 70° and 120°. At 387° the density of

[1]) F. B. JEWETT, A new Method of determining the Vapour-Density of Metallic Vapours, and an Experimental Application to the Cases of Sodium and Mercury. Phil. Mag. [6], 4, p. 546. (1902).

saturated sodium vapour is about $\frac{1}{1000}$ of that of the atmospheric air at 0° and 76 cms.

Observations.

If we now regulate the intensity of the current in the furnace in such a manner that the thermo-couple indicates a steady temperature (in many of our experiments 390°), then within the tube the density of the vapour is not everywhere the same, to be sure, for the temperature falls from the middle towards the ends, but since the surfaces of equal temperature are practically perpendicular to the beam of light, all rays pass nearly rectilinearly through the vapour. Accordingly the spectrum is only little changed; the two *D*-lines have become somewhat stronger, which we shall, for the present, ascribe to absorption by the sodium vapour in the tube.

We now blow a feeble current of air through the tube *A* which thus is slightly cooled, so that sodium condenses on it, the vapour density in its neighbourhood diminishing. We soon see the sodium lines broaden considerably. This cannot be the consequence of increased absorption, since the average vapour density has decreased a little. The reason is that rays of light with very great refractive indices are now bent towards q' (fig. 3), rays with very small indices towards q; hence in the image of the slit *P* which is formed on *S*, rays belonging to regions on both sides of the *D*-lines no longer occur, while yet this image remains perfectly sharp since the course of all other rays of the spectrum has not been perceptibly altered. If now at the same time the tube *B* is heated by a current of e.g. 20 Ampères, by which the density gradient in the space between the tubes is increased, the breadth of the lines becomes distinctly greater still. The heat generated in the tube by the current is about 1 calory per second; it is, however, for the greater part conducted away to the cooled ends of the tube, so that the rise of temperature can only be small.

By switching a current key and a cock, *A* and *B* can be made to suddenly exchange parts, so that *A* is heated, *B* cooled. The dark bands then shrink, pass into sharp *D*-lines and then expand again, until, after a few minutes, they have recovered their original breadth.

Fine and sharp, however, the lines in the transition stage are only if the temperature of the furnace is very constant. If it rises or sinks the minimal breadth appears to be not so small. In this

case, however, there certainly exist currents in the mass of vapour which cause the distribution of density to be less regular. Also when A and B are at equal temperatures, we sometimes see the sodium lines slightly broadened; it stands to reason to attribute this also to refraction in such accidental irregularities.

That spectral lines possess some breadth is commonly ascribed either to motion of the light-emitting molecules in the line of sight or to changes in the vibrational period of the electrons by the collisions of the molecules. We now have a third cause: anomalous dispersion in the absorbing medium. The whole series of phenomena, observed in our sodium tube, corroborates the opinion that this latter cause must in general be regarded as by far the most important. It will appear that this conclusion holds not only for dark but also for bright spectral lines.

If the slit in the diaphragm P is made much broader towards p', this has no influence on the spectrum as long as A and B are at the surrounding temperature. The D-lines appear as in α, Pl. I. If now A is cooled below this temperature, B raised above it, the dark D-lines only broaden in the direction of the shorter wave-lengths, while at the side of the longer wave-lengths the intensity of the light is even increased, since now also anomalously bent rays from the radiation field p' can reach the point S through the slit Q. (see β, Pl. I). The spectrum β passes into γ when the temperature difference between A and B is made to change its sign or also when the original temperature difference is maintained and the slit in P is made much broader towards p instead of towards p'. A small shifting of the whole diaphragm P (starting from the condition in which it was when taking β) so that S falls exactly in the shadow, causes the spectrum δ to appear, which makes the impression of an emission spectrum of sodium with slightly shifted lines, although it is evidently only due to rays from the field p' which have undergone anomalous dispersion in the vapour.

Let us now return to the diaphragm P with a narrow slit placed on the optical axis. (A piece of glass coated with tinfoil in which a slit was cut out, was generally used). The spectrum then shows broad bands when there is a density gradient between A and B. If beside the slit an opening is cut in the tinfoil, a group of rays of definite refractivity (and consequently also of definite wave-lengths) is given an opportunity to reach S through Q, and a bright spot is formed in the dark band, the shape of which depends on the shape of the opening in the tinfoil, but is by no means identical with it.

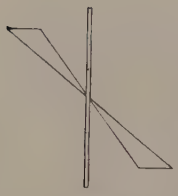

Fig. 4.

If e. g. the opening in the diaphragm has the shape of fig. 4, then the spectrum ε is obtained. When the density gradient is diminished the figure shrinks, ζ; if now the density gradient is made to change its sign and to increase, the spectrum proceeds through the stages α (gradient exactly zero) and η to θ.

The relation between the shape of the opening in the diaphragm and that of the bright spots in the spectrum might easily have been foretold from the shape of the dispersion curve. Having, however, experimentally found the relation between the two figures for a simple case as the one above, it is not difficult to design for any desired distribution of light the shape of the required opening in the diaphragm. The flower ι and its inversion $\varkappa$ required the diaphragm, represented in fig. 5. By reversing the gradient the image ι passes into $\varkappa$.

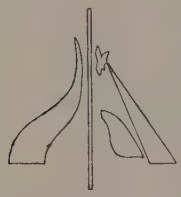

Fig. 5.

So in this way one may also arbitrarily produce duplications, reversals, bright or dark ramifications of spectral lines and it would e. g. be possible faithfully to reproduce all phenomena observed in this respect in the spectra of sun-spots, faculae or prominences. On Plate II a number of arbitrary distributions of light have been collected. They were all produced in sodium vapour of 390° on the average; α' is again the spectrum with equal temperatures of the tubes A and B. In ν on the dark dispersion band D_2 a bright double line is seen, reminding us of the spectrum of the calcium flocculi of HALE. In the same negative D_1 also shows a fine double line, which however is no longer visible in the reproduction. The spectra φ, χ, ψ imitate the origin of a sun spot and prominence spectrum; φ namely represents the spectrum of the quiet solar limb with radially placed slit; in χ a prominence appears and a spot with phenomena of reversal; ψ shows all this in a stronger degree. If now the density gradient is made to change sign, the image first shrinks again to φ after which it expands to ω, in a certain sense the inversion of ψ. The remarkable aspect of these gradual changes, admitting of perfect regulation, is only imperfectly rendered by the photographs.

The relation between the curvature of the rays and the density gradient.

The question arises whether it is *probable* that circumstances as were realised in our experiments are also met with in nature, or in

common spectroscopical investigations undertaken with entirely different purposes.

We remark in the first place that curiously shaped diaphragm openings are not absolutely essential for the production of phenomena as those described above. If e.g. our source of light had a constant, say circular shape; if on the other hand the direction and magnitude of the density gradient in our tube had not been so regular, but very different in various places of the field reproduced by the lens F, then the D-lines would also have shown all sorts of excrescences, now determined by the configuration of the density distribution.

In the second place we will try to form some idea of the quantitative relations.

The radius of curvature ϱ of the path of the most deviated rays, occurring in our photographs, may be easily estimated from the distance d of the diaphragm to the middle of the furnace, the distance δ of the most distant diaphragm openings to the optical axis, and the length l of the space in which the incurvation of the rays is brought about. For:

$$\varrho : l = d : \delta.$$

Putting δ=1 cm., d=110 cms., l=27 cms. this gives: ϱ=3000 cms. The average density Δ of the sodium vapour was in this case about $\frac{1}{1000}$ of that of the atmospheric air.

Let us see how ϱ changes with the density gradient.

We always have:

$$\varrho = \frac{n}{n'} \quad \ldots\ldots\ldots \quad (1)$$

if n represents the local index of refraction of the medium for the ray under consideration and $n' = \frac{dn}{ds}$ the change of this index per cm. in the direction of the centre of curvature. Approximately we have, for a given kind of light:

$$\frac{n-1}{\Delta} = \text{constant} = R$$

$$n = R\Delta + 1$$

$$n' = \frac{dn}{ds} = R\frac{d\Delta}{ds}$$

From this follows:

$$\varrho = \frac{R\Delta + 1}{R\frac{d\Delta}{ds}};$$

but since for rarefied gases n differs little from unity, even for the anomalously dispersed rays which we consider, $R\Delta$ may be neglected with regard to 1 and we may write

$$\varrho = \frac{1}{R\frac{d\Delta}{ds}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

For every kind of light ϱ is consequently inversely proportional to the density gradient of the vapour in the direction perpendicular to that of propagation.

An estimate of the magnitude of the density gradient existing, in our experiments, between A and B, may be obtained in two ways. It may namely be inferred from the produced difference of temperature, or from formula (2).

The temperature difference between A and B would have been pretty easy to determine thermo-electrically; up to the present, however, I had no opportunity to make the necessary arrangement. Besides, the relation between the density distribution in the space, passed by the rays, and the temperatures of A and B cannot be so very simple, since we have to deal not with two parallel planes but with tubes, from which moreover hang many drops of liquid sodium.

The second method at once gives an average value of $\frac{d\Delta}{ds}$ for the space passed by the rays. It requires a knowledge of $R = \frac{n-1}{\Delta}$ for a kind of ray for which in our experiments also ϱ has been determined.

Now Wood (Phil. Mag. [6], 8, p. 319) gives a table for the values of n for rays from the immediate vicinity of the D-lines. These data, however, refer to saturated sodium vapour of 644°; but we may deduce from them the values of n for vapour of 390° by means of the table which he gives in his paper on page 317.

For, when we heat from 389° to 508°, the refractive power of the vapour (measured by the number of passing interference fringes of helium light $\lambda = 5875$) becomes $\frac{98}{9} = 11$ times greater, and at further heating from 508° to 644° again $\frac{50}{4} = 12{,}5$ times greater (now found by interference measurement with light from the mercury line $\lambda = 5461$); hence from 390° to 644° the refractive power increases in ratio of 1 to $11 \times 12{,}5 = 137$.

Since now for rays, situated at 0,4 Ångström-unit from the D-

lines [1]) we have $n-1=\pm 0.36$, (as the average of three values taken from Wood's table on page 319), we ought to have with sodium vapour at 390° for the same kind of rays

$$n-1=\frac{0.36}{137}=0.0026.$$

The density Δ at 390° is, according Jewett, 0.0000016, hence

$$R=\frac{n-1}{\Delta}=\frac{0.0026}{0.0000016}=1600.$$

Then from formula (2) follows

$$\frac{d\Delta}{ds}=\frac{1}{R\varrho}=\frac{1}{1600\times 3000}=0{,}0000002.$$

Dispersion bands in the spectra of terrestrial sources.

It is very probable that, when metals evaporate in the electric arc, values of the density gradient are found in the neighbourhood of the carbons that are more than a thousand times greater than the feeble density gradient in our tube with rarefied sodium vapour [2]).

The radius of curvature will, therefore, in these cases be over a thousand times smaller than 30 meters and so may be no more than a few centimetres or even less. A short path through the vapour mass is then already sufficient to alter the direction of certain rays very perceptibly.

If now an image of the carbon points is produced on the slit of a spectroscope, then this is a *pure* image only as far as it is formed by rays that have been little refracted in the arc, but the rays which undergo anomalous dispersion do not contribute to it. Light of this latter kind, coming from the crater, may be lacking in the image of the crater and on the other hand penetrate the slit between the images of the carbon points. Thus in ordinary spectroscopic observations, not only broadening of absorption lines, but also of emission lines, must often to a considerable extent be attributed to anomalous dispersion.

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[1]) The spectrum ε in our plate shows that the extremities of the peaks correspond pretty well to light of this wave-length; for they approach the D-lines to a distance which certainly is no more than $^1/_{15}$ of the distance of the D-lines which amounts to 6 Ångstr.-units. For these rays the opening of the diaphragm was 1 cm. distant from the optical axis.

[2]) If we e. g. put the vapour density of the metal in the crater, where it boils, at 0.001, the density of the vapour outside the arc at a distance of 1 cm. from the crater, at 0.0001, then we have already an average gradient 5000 times as large as that used in our experiments.

When we bear this in mind, many until now mysterious phenomena will find a ready explanation. So e.g. the fact that LIVEING and DEWAR[1]) saw the sodium lines strongly broadened each time when vapour was vividly developed after bringing in fresh material, but saw them become narrower again when the mass came to rest, although the density of the vapour did not diminish. If by pumping nitrogen into the evaporated space the pressure was gradually increased, the lines remained sharp; but if the pressure was suddenly released, they were broadened. All this becomes clear as soon as one has recognised in the lines dispersion bands, which must be broad when the density of the absorbing vapour is irregular, but narrow, even with dense vapour, if only the vapour is evenly spread through the space.

Another instance. According to the investigations of KAYSER and RUNGE the lines, belonging to the second secondary series in the spectra of magnesium, calcium, cadmium, zinc, mercury, are always hazy towards the red and are sharply bordered towards the violet, whereas lines, belonging to the first secondary series or to other series are often distinctly more widened towards the violet. With regard to the spectrum of magnesium they say:[2]) "Auffallend ist bei mehreren Linien, die wir nach Roth verbreitert gefunden haben, dass sie im ROWLAND'schen Atlas ganz scharf sind, und dann stets etwas kleinere Wellenlänge haben. So haben wir 4703,33, ROWLAND 4703,17; wir 5528,75, ROWLAND 5528,62. Unschärfe nach Roth verleitet ja leicht der Linie grössere Wellenlänge zuzuschreiben; so gross kann aber der Fehler nicht sein, denn die ROWLAND'sche Ablesung liegt ganz ausserhalb des Randes unserer Linie. Wir wissen daher nicht, woher diese Differenz rührt." KAYSER has later[1]) given an explanation of this fact, based on a combination of reversal with asymmetrical widening; but a more probable solution is, in my opinion, to regard the widened serial lines as dispersion bands.

If we namely assume that, when we proceed from the positive carbon point, which emits the brightest light, to the middle of the arc, the number of the particles associated with the second secondary series decreases, then rays coming from the crater and whose wavelength is slightly greater than that of the said serial lines will be curved so as to turn their concave side to the carbon point. Their origin is erroneously supposed to be in the prolongation of their final direction, so they *seem* to come from the arc, and one believes

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[1]) LIVEING and DEWAR, On the reversal of the lines of metallic vapours, Proc. Roy. Soc. 27, p. 132—136: 28, p. 367—372 (1878—1879).

[2]) KAYSER und RUNGE, Über die Spektren der Elemente, IV, S. 13.

[1]) KAYSER. Handbuch der Spektroskopie II, S. 366.

to see light emitted by the vapour, in which light different wave-lengths occur, all greater than the exact wave-length of the serial lines. The observed displaced lines of the second secondary series are consequently comparableto apparent emission lines of the spectrum *d* of our plate I.

In this explanation things have been represented as if the light of these serial lines had to be *exclusively* attributed to anomalous dispersion. Probably however in the majority of cases emission proper will indeed perceptibly contribute to the formation of the line; the sharp edge must then appear in the exact place belonging to the particular wave-length.

How can we now explain that lines of other series are diffuse at the opposite side? Also this may be explained as the result of anomalous dispersion if we assume that of the emission centres of these other series the density *increases* when we move away from the positive carbon point. In this case namely the rays originating in the crater, which are concave towards the carbon point and consequently seem to come from the arc, possess shorter wave-lengths than the serial lines, i. e. the serial lines appear widened towards the violet. This supposition is not unlikely. For the positive and negative atomic ions which according to STARK's theory are formed in the arc by the collision of negative electronic ions, move in opposite directions under the influence of the electric field; hence the density gradients will have opposite signs for the two kinds. Series whose lines are diffuse towards the red and series whose lines flow out towards the violet would, according to this conception, belong to atomic ions of opposite signs — a conclusion which at all events deserves nearer investigation.

The examples given may suffice to show that it is necessary systematically to investigate to what extent the already known spectral phenomena may be the result of anomalous dispersion. A number of cases in which the until now neglected principle of ray-curving has undoubtedly been at the root of the matter are found in KAYSER's handbook II, p. 292—298, 304, 306, 348—351, 359—361, 366.

Dispersion bands in the spectra of celestial bodies.

Since almost any peculiarity in the appearance of spectral lines may be explained by anomalous dispersion if only we are at liberty to assume the required density distributions, we must ask when applying this principle to astrophysical phenomena: can the values of the density gradient for the different absorbing gases in celestial

bodies really be such, that the rays are sufficiently curved to exert such a distinct influence on the distribution of light in the spectrum?

In former communications [1]) I showed that the sun e.g. may be conceived as a gaseous body, the constituents of which are intimately mixed, since all luminous phenomena giving the impression as if the substances occurring in the sun were separated, may be brought about in such a gaseous mixture by anomalous dispersion. We will now try to prove that not only this *may* be the case, but that it *must* be so on account of the most likely distribution of density.

Let us put the density of our atmosphere at the surface of the earth at 0.001293. At a height of 1050 cms. it is smaller by $\frac{1}{760}$ of this amount, so that the vertical density gradient is

$$\frac{0.001293}{1050 \times 760} = 16 \times 10^{-10}.$$

The horizontal gradients occurring in the vicinity of depressions are much smaller; even during storms they are only about $\frac{1}{1000}$ of the said value [2]). Over small distances the density gradient in the atmosphere may of course occasionally be larger, through local heating or other causes.

Similar considerations applied to the sun, mutatis mutandis, cannot lead however to a reliable estimate of the density gradients there occurring. A principal reason why this is for the present impossible is found in our inadequate knowledge of the magnitude of the influence, exerted by *radiation pressure* on the distribution of matter in the sun. If there were no radiation pressure, we might presuppose, as is always done, that at the level of the photosphere gravitation is 28 times as great as on the earth; but it is counteracted by radiation pressure to a degree, dependent on the size of the particles; for some particles it may even be entirely abolished. The radial density gradient must, therefore, in any case be much smaller than one might be inclined to calculate on the basis of gravitational action only.

Fortunately we possess another means for determining the radial density gradient in the photoshere, at any rate as far as the order of magnitude is concerned. According to SCHMIDT's theory the photosphere is nothing but a critical sphere the radius of which is equal

[1]) Proc. Roy. Academy Amsterdam, II, p. 575; IV, p. 195; V, p. 162, 589 and 662; VI, p. 270; VIII, p. 134, 140 and 323.

[2]) ARRHENIUS. Lehrbuch der kosmischen Physik, S. 676.

to the radius of curvature of luminous rays whose path is horizontal at a point of its surface. This radius of curvature is consequently $\varrho = 7 \times 10^{10}$ cms., a value which we may introduce into the expression for the density gradient:

$$\frac{d\Delta}{ds} = \frac{1}{R\varrho}$$

The refractive equivalent R for rays that undergo no anomalous dispersion varies with different substances, to be sure; but in an approximate calculation we may put $R = 0{,}5$. Then at the height of the critical sphere we shall have:

$$\frac{d\Delta}{ds} = \frac{1}{0.5 \times 7 \times 10^{10}} = 0.29 \times 10^{-10},$$

(this is 50 times less than the density gradient in our atmosphere). All arguments supporting SCHMIDT's explanation of the sun's limb, are at the same time in favour of this estimate of the radial density gradient in the gaseous mixture.

Let us now consider rays that do undergo anomalous dispersion. In order that e.g. light, the wave-length of which differs but very little from that of one of the sodium lines, may seem to come from points situated some arc seconds outside the sun's limb, the radius of curvature of such anomalously bent rays need only be slightly smaller than 7×10^{10} cms. Let us e.g. put

$$\varrho' = 6 \times 10^{10} \text{ c.m.}$$

If we further assume that of the kind of light under consideration the wave-length is 0.4 Ångström-units greater than that of D_1, then for this kind of light $R' = 1600$, as may be derived from the observations of WOOD and of JEWETT [1]); we thus find for the density gradient of the sodium vapour

$$\frac{d\Delta'}{ds} = \frac{1}{R'\varrho'} = \frac{1}{1600 \times 6 \times 10^{10}} = 0.0001 \times 10^{-10},$$

a quantity, 2900 times smaller than the density gradient of the gaseous mixture.

Hence if only $\frac{1}{3000}$ part of the gaseous mixture consists of sodium vapour, then, on account of the radial density gradient, the critical sphere will already seem to be surrounded by a "chromosphere" of light, this light having a striking resemblance with sodium light. This kind of light has, so to say, its own critical sphere which is larger than the critical sphere of the not anomalously refracted light. If the

1) See page 352.

percentage of sodium were larger, the "sodium chromosphere" would appear higher.

It is customary to draw conclusions from the size of the chromospheric and flash crescents, observed during a total eclipse with the prismatic camera, as to the *height* to which various vapours occur in the solar atmosphere. According to us this is an unjustified conclusion. On the other hand it will be possible to derive from these observations data concerning *the ratio in which these substances are present in the gaseous mixture*, provided that the dispersion curves of the metallic vapours, at known densities, will first have been investigated in the laboratory.

Until now we only dealt with the normal radial density gradient. By convection and vortex motion however irregularities in the density distribution arise, with gradients of various direction and magnitude. And since on the sun the resultant of gravitation and radiation pressure is relatively small, there the irregular density gradients may sooner than on the earth reach values that approach the radial gradient or are occasionally larger.

The incurvation of the rays in these irregularities must produce capriciously shaped sodium prominences, the size of which depends, among other causes, on the percentage of sodium vapour in the gaseous mixture.

So the large hydrogen and calcium prominences prove that relatively much hydrogen and calcium vapour is present in the outer parts of the sun; but perhaps even an amount of a few percents would already suffice to account for the phenomena [1]).

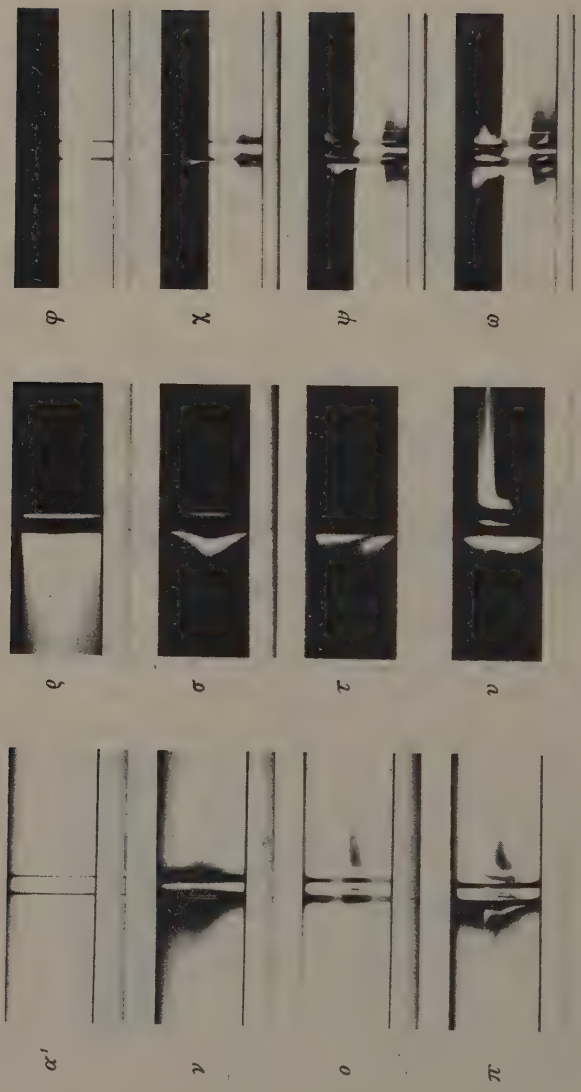
If we justly supposed that non-radially directed density gradients are of frequent occurrence in the sun, and there disturb the general radial gradient much more than on the earth, then not only rays from the marginal region but also rays from the other parts of the solar disc must sensibly deviate from the straight line. Chiefly concerned are of course the rays that undergo anomalous dispersion. *Every absorption line of the solar spectrum must consequently be enveloped in a dispersion band.*

To be sure, absorption lines of elements which in the gaseous mixture only occur in a highly rarefied condition, present themselves as almost sharp lines, since for these substances all density gradients are much smaller than for the chief constituents, and so the curvature of the rays from the vicinity of these lines becomes imperceptible.

[1]) This result would be in accordance with a hypothesis of Schmidt (Phys. Zeitschr. 4, S. 232 and 341) according to which the chief constituent of the solar atmosphere would be a very light, until now unknown gas.

Proceedings Royal Acad. Amsterdam. Vol. IX.

Heliotype, Van Leer, Amsterdam.



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Heliotype, Van Leer, Amsterdam.

Also of strongly represented elements some lines may appear sharp, since not all lines of the same element, with given density, cause anomalous dispersion in the same degree. Perhaps even there are absorption lines which under no condition give rise to this phenomenon; though this were rather improbable from the point of view of the theory of light.

Be this as it may, the mentioned limitations do not invalidate our principal conclusion: that the general interpretation of the solar spectrum has to be modified. We are obliged to see in FRAUNHOFER's lines not only absorption lines, as KIRCHHOF does, but chiefly dispersion bands (or dispersion lines). And that also on the distribution of light in the stellar spectra refraction has a preponderant influence, cannot be doubted either.

We must become familiar with the idea that in the neighbourhood of the celestial bodies the rays of light are in general curved, and that consequently the whole interstellar space is filled with *non-homogeneous radiation fields* [1]) of different structure for the various kinds of light.

Chemistry. — "*On a substance which possesses numerous* [2]) *different liquid phases of which three at least are stable in regard to the isotropous liquid.*" By Dr. F. M. JAEGER. (Communicated by Prof. H. W. BAKHUIS ROOZEBOOM).

§ 1. The compound which exhibits the highly remarkable phenomena to be described, is *cholesteryl-cinnamylate*: $C_{27}H_{45}O_2C.CH:CHC_6H_5$.

I have prepared this substance by melting together equal quantities of pure cholesterol and cinnamyl-chloride in a small flask, which was heated for about two hours in an oilbath at 190°. It is of the greatest importance, not to exceed this temperature and the time of heating, as otherwise the liquid mass, which commences to darken, even under these conditions, yields instead of the desired derivative a brown resin which in solution exhibits a green fluorescence.

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[1]) Das ungleichmässige Strahlungsfeld und die Dispersionsbanden. Physik. Zeitschr. 6, S. 239—248, 1905.

[2]) In the Dutch publication, I have said: five. Since that time however, more extended microscopical observation has taught me, that probably there are an infinite number of anisotropous liquid phases, no sharply fixed transition being observed in this manner. The hypothesis, that the transition of the first anisotropous liquid phase into the isotropous should be continuous, would therefore be made more probable in this way. However there are observed some irreversibilities by passing from solid to liquid state and vice-versa, which yet I cannot explain at this moment.

The solidified mass is dissolved in boiling ether, and the brown liquid is boiled with animal charcoal for an hour in a reflux-apparatus. To the filtrate is then added absolute alcohol, heated to 40°, until the liquid gets turbid. On being set aside for a few hours the ester deposits in small, almost white glittering leaflets. These are collected at the pump, washed with a little ether-alcohol, and then recrystallised several times from boiling ethyl acetate, to which each time some alcohol may be added to precipitate the bulk of the ester. The pure, quite colourless, beautifully crystallised compound shows no heterogenous components under the microscope.

§ 2. The following experiments were carried out in the usual manner; the substance was contained in small, thin-walled testtubes, whilst surrounded of a cylindrical air-bath, and whilst the thermometer was placed in the liquid completely which covered the mercury reservoir. The temperature of the oilbath was gradually raised with constant stirring and now the following facts were noticed.

At about 151° the solid mass begins to soften [1]) while brilliant colours appear here and there at the sides, principally green and violet, with transmitted light the complimentary colours red and yellow. At about 157° the mass is a thick fluid and strongly doubly refracting; the ground tone of the phase is orange-red, whilst, on stirring with the thermometer, the liquid crystals everywhere form links of lustrous bright green and violet slides. Afterwards, by the construction of the cooling curve, I determined sharply the temperature t [155°.8 C.] at which the substance solidifies; the break in the curve is distinct as the heat effect is relatively large and the undercooling was prevented by inoculation with a solid particle of the ester.

The colour of the liquid phase is now but little changed on further heating; on the other hand its consistency becomes gradually more and more that of a thin liquid. At 199°.5 it is nearly colourless and one would expect it to become presently quite clear.

But at that temperature the mass becomes all of a sudden enamel-white, and rapidly thickens, while still remaining doubly-refracting. We now observe plainly a separation into two liquid layers which are here both anisotropous. The interference colours have now totally disappeared. Then, on heating slowly, the liquid phase becomes isotropous at 201.3° and quite clear. The isotropous liquid is colourless.

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[1]) Bondzynski and Humnicki (Zeitsehr. f. physiol. Chem. **22**, 396, (1896), describe a cinnamylate which as regards solubility etc. agrees with mine, but which melts at 149°. This is evidently identical with my first temperature of transition.

On cooling, the following phenomena occur: At about 200° the isotropous liquid becomes turbid, at 198° the doubly-refracting mass attains its greatest viscosity; at 196° it has already become thinner, but now at about 198° it again becomes thicker and the whole appearance of the phase is strikingly altered, although still remaining doubly refracting. It then seems to pass gradually into the green and red coloured, doubly refracting liquid phase, which, if we prevent the undercooliug by inoculation, solidifies at 155°.8.

If the solid substance is melted under LEHMANN's crystallisation-microscope, — where the conditions of experimenting are naturally quite others than before, — it seems, that but *one* liquid phase, the green and red coloured, is *continually* changed into the isotropous one: no sudden changing is observed. On cooling, the aspect of the anisotropous phase now obtained, is quite different from the first mentioned.

I also think I must come to the conclusion that the liquid phase (t = about 190°) occurring on cooling is perhaps only the passage to the other three, so that here, *three stable* liquid phases might occur. It is very remarkable that the transitions of the two stable anisotropous phases into the intermediate one appear, when we work carefully, quite continuous; the viscosity appears to pass gradually into that of the more stable phases. Remarkable also is the impossibility to find the temperatures of transition exactly the same on the rising, or falling, temperature of the external bath. The values obtained for the initial and final temperature of each phase-traject vary within narrow limits. The same is the case when, on melting the solid substance, one wishes to determine the point where the first softening of the mass takes place; in the determination of the temperature, intervals such as from 147° to 156° are noticed. The progressive change of the cooling of isotropous-liquid to solid resembles here in a high degree a process where a continuous transition exists between the different stadia. It is as if the labile phase is composed of an entire series of conditions which occur successively to form the connection on one side between anisotropous and isotropous-liquid. The whole shows much resemblance to a gradual dissociation and association between more or less complicated molecule-complexes. It is quite possible that the transitions solid-liquid occur really *continuously* instead of *suddenly*, in which case an uninterrupted series of labile intermediate conditions — which cannot be realised in most substances — are passed, some of which intermediate conditions might be occasionally fixed in those substances which like these chole-

steryl-esters usually display the phenomena of the doubly-refracting liquid-conditions. All this seems probable to the investigator, the more so as it has been proved by LEHMANN, that in my other cholesteryl-esters, even in the case of the caprinate, *both* or one of the two anisotropous liquid-phases were always labile and only realizable on undercooling; some of them, such as the *iso*butyrate, only exhibited their labile anisotropous liquid-phases, when containing some impurities and not when in a pure condition. With the idea of a gradual dissociation of compound molecule-complexes into more simple ones, agrees the fact that the anisotropous liquid phases have never been known yet to occur *after* the isotropous ones; this is always the end-phenomenon, which is accounted for by the fact that a dissociation of this kind is always found to increase with a rise of temperature.

That the cooling between solid and anisotropous-liquid does not proceed so suddenly as may be predicted from the great calorific effect is shown in the case of the cinnamylate from the fact that, after the solidification, particularly at the side of the test tubes, the interference-colours, which are characteristic before the transition of the phases into each other, remain visible for a very long time, often many hours, then slowly disappear. Even with great enlargement, no well defined crystals can be discovered in those coloured parts; the whole conveys the impression of a doubly-refracting, irregular network of solidified liquid droplets, just like the liquid crystals which present themselves to the eye with the aid af a powerful enlargement [1]).

In these obscure phenomena we are bound to notice the more or less labile and partially realized intermediate stadia in a continuous transition liquid $\leftrightarrows$ solid. The view expressed by LEHMANN, that there should be present a difference between the kinds of molecules in the different aggregate conditions, is adopted here with this difference, that such a difference of association of the molecules is thought quite compatible with the phenomenon of the *continuity* of the aggregate conditions, treated of here.

§ 3. I wish to observe, finally, that cholesteryl-cinnamylate when subjected frequently to these melting experiments, soon undergoes a small but gradually increasing decomposition, which shows itself in the yellow colour of the mass and the fall of the characteristic temperature-limits.

Zaandam, 26 Oct. '06.

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[1]) A still more distinct case of this phenomenon has now been found by me in *α-phytosteryl-propionate*, which I hope soon to discuss in another communication.

Chemistry. *"The behaviour of the halogens towards each other".* By Prof. H. W. BAKHUIS ROOZEBOOM.

If the phase-doctrine in its first period was concerned mainly with the question whether two or more substances in the solid condition give rise to chemical compounds, or mixed crystals, or remain unchanged in the presence of each other, lately it has commenced to draw conclusions from the form of the melting point lines of the solid mixtures, both for the nature of those solid mixtures and of the liquid mixtures into which they pass, namely whether, and to what extent, compounds occur therein:

Likewise, the same questions may be answered in regard to liquid and vapour from the equilibrium lines for those two phases, namely boiling point lines or vapour pressure lines.

The three systems of the best known halogens having now been investigated their mutual behaviour may be surveyed.

As regards chlorine and iodine, STORTENBEKER had proved in 1888 that no other compounds occur in the solid condition but ICl_3 and ICl. He also showed that it is probable that ICl, on melting, liquefies to a very large extent without dissociation, whilst on the other hand ICl_3 is almost entirely dissociated into $ICl + Cl_2$.

Miss KARSTEN has now added to this research by the determination of the boiling point lines. This showed that the liquid and the vapour line approach each other so closely in the vicinity of the composition ICl [1]), that the conclusion must be drawn that the dissociation of ICl is also exceedingly small in the vapour, it being already known that it is very large in the case of ICl_3.

From the investigation of MEERUM TERWOGT [2]) it has been shown that Br and I form only one compound BrI which in the solid state forms mixed crystals both with Br and I and which on account of the form of the vapour pressure and boiling point lines is largely dissociated in the liquid and gaseous states.

Finally it now appears from an investigation by Miss KARSTEN that Chlorine and Bromine only give mixed crystals on cooling and that in a connected series, whilst, in agreement with this no indication for the existence of the compound in the liquid or vapour could be deduced from the form of the boiling point line.

We, therefore come to the conclusion that ICl_3 is a feeble and ICl a strong compound. IBr is also a feeble compound and no compound exists between Cl and Br. The combining power is, therefore,

[1]) Still closer than represented in Fig. 7, p. 540. These proceedings [VIII] 1904.

[2]) These proceedings VI, p. 331.

greatest in the most distant elements and greater in Br + I than in Br + Cl.

From the researches of MOISSAN and others it follows that Fluorine yields the compound IF_5 which is stable even in the vapour-condition. With Bromine, the compound BrF_3 is formed but no compound is formed with Chlorine. This, also, is in harmony with the above result.

As, however, the compounds with Fluorine have not been studied from the standpoint of the phase-doctrine, there does not exist as yet a reasonable certainty as to their number or their stability.

Mathematics. — "*Second communication on the* PLÜCKER *equivalents of a cyclic point of a twisted curve.*" By Dr. W. A. VERSLUYS. (Communicated by Prof. P. H. SCHOUTE).

§ 1. If the origin of coordinates is a cyclic point (n, r, m) of a twisted curve C the coordinates of a point of C lying in the vicinity of the origin on a branch passing through the origin can be represented as follows:

$$x = a\,t^n,$$
$$y = b_0\,t^{n+r} + b_1\,t^{n+r+1} + b_2\,t^{n+r+2} + \text{etc.},$$
$$z = c_0\,t^{n+r+m} + c_1\,t^{n+r+m+1} + c_2\,t^{n+r+m+2} + \text{etc.}$$

Let q_1 be the greatest common divisor of n and r, let q_2 be that of r and m, q_3 that of m and $n + r$ and finally q_4 that of n and $r + m$.

If $q_1 = q_2 = q_3 = q_4 = 1$ the PLÜCKER equivalents depend only on n, r and m. In a preceding communication[1]) I gave the PLÜCKER equivalents for this special case[2]).

§ 2. If the 4 G. C. Divisors q are not all unity, the PLÜCKER equivalents of the cyclic point (n, r, m) depend on the values of the coefficients b and c, just as in general for a cyclic point of a plane curve given by the developments:

$$x = t^n,$$
$$y = t^{n+m} + d_1\,t^{n+m+1} + d_2\,t^{n+m+2} + \text{etc.},$$

the vanishing of coefficients d influences the number of nodal points and double tangents equivalent to the cyclic point (n, m)[3]).

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[1]) Proceedings Royal Acad. Amsterdam, Nov. 1905.

[2]) The deduction of these equivalents is to be found among others in my treatise: "*Points sing. des courbes gauches données par les équations:* $x = t^n$, $y = t^{n+r}$, $z = t^{n+r+m}$," inserted in "*Archives du Musée Teyler*", série II, t. X, 1906.

[3]) A. BRILL and M. NOETHER. Die Entwicklung der Theorie der algebraischen Functionen, p. 400. *Jahresbericht der Deutschen Mathematiker-Vereinigung*, III, 1892—93.

If the coefficients c and b are not zero, if no special relations exist between these coefficients and if besides n, r and m are greater than *one*, the cyclic point (n, r, m) is equivalent to

$$n-1 \text{ stationary points } \beta \text{ and to}$$

$$\{(n-1)(n+r-3)+q_1-1\} : 2 \text{ nodes } H.$$

The osculating plane of the curve C in the cyclic point (n, r, m) is equivalent to

$$m-1 \text{ stationary planes } \alpha \text{ and to}$$

$$\{(m-1)(r+m-3)+q_2-1\} : 2 \text{ double planes } G.$$

The tangent of the curve C in the cyclic point (n, r, m) is equivalent to

$$r-1 \text{ stationary tangents } \theta, \text{ to}$$

$$\{(r-1)(n+r-3)+q_1-1\} : 2 \text{ double tangents } \omega \text{ and to}$$

$$\{(r-1)(r+m-3)+q_2-1\} : 2 \text{ double generatrices } \omega' \text{ of the developable } O \text{ formed by the tangents of the curve } C.$$

§ 3. The cyclic point (n, r, m) of the curve C is an $n+r$-fold point of the developable O of which C is the cuspidal curve.

The cyclic point (n, r, m) counts for

$$(n+r-2)(n+r+m)$$

points of intersection of the cuspidal curve C with the second polar surface of O for an arbitrary point.

Through the cyclic point (n, r, m) of the cuspidal curve C pass

$$\{n(n+2r+m-4)+q_3-q_2\} : 2$$

branches of the nodal curve of the developable O.

All these nodal branches touch in the cyclic point (n, r, m) the tangent of the cuspidal curve C (the x-axis).

They have with this common tangent in the point of contact

$$\{(n+r)(n+2r+m-4)+q_4-q_2\} : 2$$

points in common.

The nodal branches passing through the cyclic point (n, r, m) all have in this point as osculating plane the osculating plane $z=0$ of the cuspidal curve C.

These nodal branches have with their osculating plane $z=0$ in the cyclic point (n, r, m)

$$\{(n+r+m)(n+2r+m-4)+q_1-q_2\} : 2$$

points in common.

§ 4. The case of an ordinary stationary plane α, the point of contact of which is a cyclic point $(1, 1, 2)$, shows that through a

cyclic point branches of the nodal curve can pass not touching in this point the cuspidal curve.

These intersecting nodal branches exist only when $q_3 > 1$. If $r > 1$ the coefficients b and c must satisfy special conditions. If $r = 1$ then through the cyclic point (n, r, m) of the cuspidal curve pass either $q_3 : 2$, or $(q_3 - 1) : 2$ of these nodal intersecting branches. All intersecting nodal branches have a common tangent in the plane $z = 0$ if $r = 1$.

§ 5. The case of an ordinary stationary point β $(2, 1, 1)$ shows that through a cyclic point of the cuspidal curve nodal branches can pass which have the same tangent, but not the same osculating plane as the cuspidal curve. These particular nodal branches exist only when $q_4 > 1$. If $q_4 > 1$ and $m = 1$ these particular nodal branches are always present. If $q_4 > 1$ and also $m > 1$ the coefficients b and c must satisfy special conditions. These particular nodal branches have in the cyclic point (n, r, m) a common osculating plane (differing from the plane $z = 0$) if $m = 1$.

§ 6. The tangent to C in the cyclic point (n, r, m) is an r-fold generatrix g on the developable O. The r sheets of the surface O passing through the generatrix g all touch the osculating plane $z = 0$ of C in the point (n, r, m).

The generatrix g moreover meets in $\varrho - (n + 2r + m)$ points R a sheet of the surface O, when O is of order ϱ.

In every point R the generatrix g meets r branches of the nodal curve. These r branches form, when $m > r$ a singularity $(r, r, m-r)$ and the osculating plane of these nodal branches is the tangent plane of O along g.

If $m < r$ these r nodal branches form a singularity $(r, m, r - m)$ and the osculating plane of these r nodal branches is the tangent plane of O along the generatrix intersecting g in R.

If $r = m$ these r nodal branches form a singularity $(r, r, 1)$.

§ 7. In general the singular generatrix g will meet only nodal branches in the cyclic point (n, r, m) and in the points R. If $q_3 > 1$ the generatrix g may meet moreover nodal branches arising from the fact that some of the r sheets, which touch each other along g penetrate each other. These nodal branches meet g in the same point Q. If $q_3 > 1$ and $n = 1$ there is always such a point of intersection Q. If $q_3 > 1$ and $n > 1$ the coefficients b and c must satisfy some special conditions if the sheets passing through g are to penetrate each other.

Physics. — "*On the measurement of very low temperatures.* XIII. *Determinations with the hydrogen thermometer.*" By Prof. H. KAMERLINGH ONNES and C. BRAAK. (Communication N° 95^e from the Physical Laboratory of Leiden).

§ 1. *Introduction.*

The results of determinations of low temperatures made with the hydrogen thermometer, which was described in Comm. N°. 27 (June 1896) and more fully discussed in Comm. N°. 60 (September 1900), have already frequently been used, but no further particulars have as yet been given about these determinations themselves. We give them now in connection with a series of observations made in 1905 and 1906. They have served for the investigations described in Comm. N°s 95^a and 95^e (June 1906) and further for determinations of isotherms of hydrogen at low temperatures, which will be discussed in a following communication. Comprising also measurements on liquid hydrogen, they extend over the whole of the accessible area of the lower temperatures. All the precautions which proved necessary in former years, have been taken. The temperature of the bath, in which the thermometer was immerged, could be kept constant to 0°.01 at all temperatures. It was therefore to be expected, that the accuracy and reliability aimed at in the arrangement of the thermometers, might to a great extent be reached.[1]) In how far this is really the case, the following data may show.

§ 2. *Arrangement of the thermometer.*

There is little to add to Comm. N°. 60. The steel capillary connecting the thermometer bulb and the manometer, was protected from breaking by passing a steel wire along it, the ends of which are soldered to copper hoods, which may be slidden on the steel pieces *c* and *e* (Plate II, Comm. N°. 27) at the end of the capillary. The dimensions of the thermometer reservoir of Comm. N°. 60 (80 c.M^3.) did not present any difficulty in our measurements, the bath in the cryostats (see Comm. N°s. 83, 94^c, 94^d and 94^f (May and June 1905 and June 1906)) offering sufficient room besides for the thermometer and other measuring apparatus, for the stirring apparatus, which works so thoroughly, that no variation of tempe-

[1]) A complete example of the determination of very low temperatures with the hydrogen thermometer was as yet not found in the literature. Such an example follows here.

rature could be found [1]) at least with the thermo-element [2]). The section of the glass capillary forming the stem of the thermometer was 0,0779 mM². With regard to the temperature correction (see § 4, conclusion), it is desirable that this section is small. It appears both from calculation and from observation [3]), that the equilibrium of pressure between the space near the steelpoint and reservoir is still very quickly reached with these dimensions of the capillary [4]), much more quickly than the equilibrium of the mercury in the two legs of the manometer, which is inter alia also confirmed by the rapidity with which the thermometer follows fluctuations in the temperature of the bath [5]).

The determination of the pressure which is exerted on the gas, may, when the determination applies to very low temperatures, be simplified and facilitated by following the example of CHAPPUIS [6]) and making the manometer tube serve at the same time as barometer tube. The modification applied for this purpose to the arrangement according to Comm. N°. 60 Pl. VI, is represented on Pl. I, which must be substituted for part of Pl. VI belonging to Comm. N°. 60. By means of an india-rubber tube and a T-piece l_0 the thermometer (a, b, c, d, e, h, k) is connected on one side with the manometer l_1, to which (see Pl. VI Comm. N°. 60) at m_1 the reservoir at constant temperature is attached and at m_2 the barometer, on the other side with the barometer tube (n_0, n_1 (airtrap) n_2). Besides from the manometer and the barometer joined at m_2, the pressure can now also immediately be read from the difference in level of the mercury in n_2 and in g. We have not availed ourselves of this means for the determinations discussed in this Communication.

§ 3. *The hydrogen.*

The filling took place in two different ways:

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[1]) TRAVERS, SENTER and JAQUEROD, (Phil. Trans. Series A, Vol. 200, Part. II, § 6) who met with greater difficulties when trying to keep the temperature constant in their measurements, had to prefer a smaller reservoir.

[2]) A resistance thermometer is more sensible (Cf. Comm. Nos. 95ᵃ and 95ᶜ). As soon as one of suitable dimensions will be ready, the experiment will be repeated.

[3]) Calculation teaches that for reducing a pressure difference of 1 c.M. to one of 0.01 m.M., the gas flowing through the capillary requires 0.1 sec., the mercury in the manometer 4 sec. Experiment gives for this time 25 sec. This higher amount must be due to the influence of the narrowing at the glass cock k.

[4]) We must be very careful that no narrowings occur.

[5]) A great deal of time must be given to exhausting the reservoir with the mercury airpump when filling it, as the equilibrium of reservoir and pump is established much more slowly than that between dead space and reservoir.

[6]) Travaux et Mémoires du Bureau International, Tome VI.

a. By means of hydrogen prepared in the apparatus of Comm. N°. 27 with the improvements described in Comm. N°. 94*e* (June 1905) § 2. After having beforehand ascertained whether all junctures of the apparatus closed perfectly, we maintained moreover all the time an excess of pressure in the generator, in order to exclude any impurity from the gas. The expulsion of the air originally present in the apparatus was continued till it could be present in the gas at the utmost to an amount of 0.000001.

b. By means of hydrogen prepared as described in Comm. N°. 94*f* XIV. In order to apply this more effective mode of preparation, we must have liquid hydrogen at our disposal. [1]) In § 7 the equivalence of the first method with the last is demonstrated for measurements down to — 217°. It is still to be examined whether systematic errors may result from the application of the first method of filling, in measurements on liquid hydrogen by the deposition of impurities, less volatile than hydrogen. [2])

§ 4. *The measurements.*

The zero point of the thermometer is determined before and after every set of observations. Both for the zero point and for every determination of temperature, an average value is derived from three or four observations. Each of these observations consists of a reading of the barometer, preceded and followed by a reading of the manometer. The thermometers, indicating the temperature of the mercury, of the scale and of the gas in the manometer spaces are read at the beginning and at the end of every observation. The temperature of the room is kept as constant and uniform as possible.

The temperature of the thermometer reservoir is taken equal to that of the bath. This is permissible for the cryostats described in Comm. N°. 94*d* and Comm. N°. 94*f* and the treatment given there.

The temperature of the bath is kept constant by means of the resistance thermometer, described in Comm. N°. 95*c*. In order to facilitate the survey of the observations, the resistance was adjusted as accurately to the same value as possible, and by means of signals the pressure in the cryostat was regulated in such a way, that in the very sensible galvanometer the mirror made only slight oscillations about

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[1]) The hydrogen in the vacuum glass *B* (see Comm. N°. 94*f* XIV, fig. 4) proved to evaporate so slowly, that a period of two hours was left for filling and exhausting the thermometer again, which previously had been kept exhausted for a long time, being heated during part of the time (cf. footnote 5 § 2).

[2]) In a former set of observations deviations were found, which in conjunction with each other prove that the hydrogen must have been mixed with air.

its position of equilibrium. So far as it proved necessary, a correction curve was plotted of these oscillations (see Plate III Comm. N°. 83, December 1902). As a rule, however, these deviations were so slight, that they could be neglected.

Part of the capillary glass stem of the thermometer has the same temperature as the liquid bath. The length of this part is derived from the indications of a float [1]) in the cryostat, which is omitted in the drawings, not to render them indistinct. In order to find the distribution of temperature in the other parts of the capillary within the cryostat, special determinations are made, viz. *a.* by means of a resistance thermometer placed by the side of the capillary, (see Comm. N°. 83 Plate II [2])), *b.* with the aid of a thermoelement, whose place of contact was put at different heights in the cryostat, the distribution of temperature in the cryostat was examined for the case that liquid air, liquid ethylene or liquid hydrogen was used as bath, and finally *c.* the distribution at other temperatures of the bath was derived from this distribution. This may be deemed sufficient, as the volume, the temperature of which is determined, amounts only to $\frac{1}{3000}$ of the reservoir, and as an error of 50° in the mean temperature of the capillary corresponds to only 0°.01 in the temperature of the bath, while the agreement of the observations sub *a* and *b* show that an error of more than 20° is excluded.

§ 5. *Calculation of the temperatures.*

The calculation of the zero point is made by reducing the observed pressure of the gas to that under fixed circumstances, the same as taken in Comm. N°. 60. Put:

V_0 the volume of the reservoir at 0°.

u_1 the volume of that part of the glass capillary that has the same temperature t as the reservoir. As such is considered the part immerged in the liquid bath, to which is added 2 cm. of the part immediately above it.

u_2' and u_2'' the volumes of the parts of the glass capillary in the cryostat outside the bath at temperatures t_2' and t_2''.

u_3 the volume of the part of the glass capillary outside the cryostat (u_2''') and of the steel capillary at the temperature t_3.

[1]) For determinations on liquid hydrogen no float was used. The level of the liquid in the bath was derived from the volume of the evaporated gas.

[2]) The lowest part from ζ_{11} to ζ_{12} with close windings is 9 cm.. the part where the windings are farther apart (about 20 cm.) reaches up to in the top of the cryostat.

u_4 the volume at the steel point of the volumenometer.

β_1 and β_2 the variation of the volume V_0 caused by the pressure of the gas.

If H_T is the observed pressure, and H_0 and u have the same meaning as in Comm. N°. 60, the temperature is found from the formula:

$$H_T\left[\frac{V_0(1+k_1t+k_2t^2)+\beta_1+u_1}{1+\alpha t}+\frac{u_2'}{1+\alpha t_2'}+\frac{u_2''}{1+\alpha t_2''}+\frac{u_3}{1+\alpha t_3}+\frac{u_4}{1+\alpha t_4}\right]=$$

$$=H_0\left[V_0+\beta_2+u_1+u_2'+u_2''+u_2'''+\frac{u}{1+15\alpha}\right] \quad . \quad . \quad (1)$$

The change of volume of the glass stem caused by the change of temperature need not be taken into account, as little as that of u. That of the thermometer reservoir has been calculated by means of a quadratic formula, of which the coefficients k_1 and k_2 have the following values: $k_1 = 23.43 \times 10^{-6}$, $k_2 = 0.0272 \times 10^{-6}$ [1]).

Put

$$\left.\begin{aligned}\frac{u_2'}{1+\alpha t_2'}+\frac{u_2''}{1+\alpha t_2''}+\frac{u_3}{1+\alpha t_3}+\frac{u_4}{1+\alpha t_4}&=\Sigma\frac{u}{1+\alpha t}\\ H_0\left[v_0+\beta_2+u_1+u_2'+u_2''+u_2'''+\frac{u}{1+15\alpha}\right]&=A,\end{aligned}\right\} \quad . \quad (2)$$

then follows from the above for the temperature:

$$t=\frac{V_0+\beta_1+u_1-\frac{A}{H_T}+\Sigma\frac{u}{1+\alpha t}+V_0k_2t^2}{\left\{\frac{A}{H_T}-\Sigma\frac{u}{1+\alpha t}\right\}\alpha-V_0k_1} \quad . \quad . \quad . \quad (3)$$

If the term with t^2 is omitted, we find an approximate value for the temperature. Now t may be calculated again, while in the term with t^2 this value is substituted. This approximate calculation is quite sufficient.

§ 6. *Survey of a measurement.*

The observations communicated in this §, yielded the temperature corresponding to the electromotive force of the thermoelement determined in Table IV and V of Comm. 95[a] and corresponding to the resistance measured in the observation given in Table I of Comm. N°. 95[c] (in the last case even almost simultaneous).

[1]) These values have been derived from Comm. N°. 95[b]. They refer to the determinations made in 1903 on the expansion of glass. If we calculate the temperatures by means of the quadratic and cubic formula derived in the same Comm. from the observations of 1905, we find but slight differences, which amount respectively to — 0°.014 and — 0°.016 at — 100°, and remain always below 0°.01 at — 200° and lower.

The tables I and II are analogous to those of Comm. N°. 60, only column *K* has been added to the former, in which the readings from the kathetometer scale are noted down. In every measurement they are always determined, in order to be used, if necessary, as a control for the readings by means of the standard scale, in connection with the collimation differences of the telescopes.

TABLE I.

DETERMINATION IN A BATH OF LIQUID HYDROGEN. (ABOUT — 253°). READINGS.

May 5, '06, 3.10-3.30		*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*K*
	Point	14.75	7.9	974 975	20.17 17.86	9.1 8.1				
Manometer	lower top	23.00	9.0	297	22.02	9.3	15.5	15.5	15.4	
	meniscus rim	26.01	9.0	298	19.90	10.3		15.6	15.1	21.926
							15.5	14.3		
	higher top	15.10	7.9	974	20.17	9.1				
	meniscus rim	18.43	7.9	975	17.86	8.1	15.5			89.294
Barometer	lower top	21.03	8.7	297	22.02	9.3	15.5			
	meniscus rim	22.70	8.6	298	19.90	10.3	15.5			
	higher top	25.82	9.6	1058	28.07	11.3	15.7			103.279
	meniscus rim	27.98	10.0	1059	25.43	11.0	15.7			
Manometer	lower top	22.98	9.3	297	22.02	9.3	15.4	15.5	15.4	
	meniscus rim	25.99	9.3	298	19.90	10.3		15.4	15.2	
							15.3	14.3		
	higher top	15.06	7.9	974	20.17	9.1				
	meniscus rim	18.42	8.0	975	17.86	8.1	15.5			
	Point	14.76	8.0	974 975	20.17 17.86	9.1 8.1				

TABLE II.

DETERMINATION IN A BATH OF LIQUID HYDROGEN (ABOUT — 253°). CORRECTED AND CALCULATED DATA OF THE OBSERVATION.

		A'	B'	C'	D'	E'	F'
Manometer	lower meniscus	296.55	296.70	14.8	14.9		
	height	1.39		14.9	14.8		
	higher meniscus	976.21	976.37	14.9	13.8	81.53	0.14
	height	1.46					
Barometer	lower meniscus	297.46	297.48	14.8			
	height	0.77					
	higher meniscus	1058.87	1058.90	15.0			
	height	0.83					

The correction was applied for the difference in level of barometer and manometer (cf. also Comm. N°. 60). In this way we find H_T, the pressure of the gas in the thermometer.

TABLE III.

DETERMINATION IN A BATH OF LIQUID HYDROGEN. (ABOUT —253°). DATA FOR THE CALCULATION,

$u_1 = 0.0105$ cm³	
$u_2' = 0.0126$ »	$t'_2 = -162°$
$u_2'' = 0.0140$ »	$t''_2 = 0°$
$u_3 = 0.6990$ »	$t_3 = 14°.5$
$u_4 = 0.2320$ »	$t_4 = 14°.9$
$u_2''' = 0.1141$ »	
$H_T = 81.53$ m.m.	
$V_0 = 82.265$ cm³	
$\beta_1 = -0.0041$ » , $\beta_2 = +0.0021$ cm³	
$H_0 = 1091.88$ mm.	
$u = 0.7991$ cm³	

From the indication of the float the value of u_1 is found. u_2' and u_2'' are chosen such that the circumstances are as closely as possible equal to those for which the distribution of temperature in the cryostat is determined. We get now the table III, in which H_0 is the zero point pressure.

From these data with formula (3), where the value 0,0036627 of Comm. N°. 60 [1]) was assumed for α, follows for the approximate value of the temperature:

$$t = -252°.964$$

and after application of the correction for the quadratic term:

$$t = -252°.964 + 0°.035 = -252°.93.$$

§ 7. *Accuracy of the determinations of the temperature.*

In order to arrive at an opinion about the error of the observations with the hydrogen thermometer, we determine the differences of the hydrogen temperatures found in different observations in which the resistance was adjusted to the same value, reduction having been applied for small differences left.

The mean error of a single determination derived from the differences of the readings of the thermometer, which succeed each other immediately, is on an average $\pm 0°.0074$, from which we derive for the mean error of a temperature $\pm 0°.0043$, assuming that on an average 3 observations have served to determine a temperature. As a rule no greater deviations than 0°.02 were found between the separate readings of one determination. Only once, on Oct. 27th '05 (cf. Comm. N°. 95c Tab. I) a difference of 0°.04 occurred. Even at the lowest temperatures only slight deviations occur. Thus on May 5th '06 two of the observations in the neighbourhood of the boiling point of hydrogen (cf. Comm. 95a Tab. VI,, observation N°. 30, and Comm. N°. 95c Tab. I) yielded:

$3^u 20'$	$-252°.926$
$3^u 58'$	$-252°.929$

the two others with another resistance:

$2^u 35'$	$-252°.875$
$3^u\ 7'$	$-252°.866$ [2])

Determinations of one and the same temperature on different days

[1]) From the values of α found by CHAPPUIS at different pressures and from BERTHELOT'S calculations follows by extrapolation from CHAPPUIS' value for $p = 1000$ m M. $\alpha = 0.00366262$ for $p = 1090$ m.M., from TRAVERS' value of α for 700 m.M. with the same data $\alpha = 0.00366288$ for $p = 1090$ m.M.

[2]) At both these temperatures the indications of the resistance thermometer were not made use of, but only the pressure in the cryostat was kept constant. That in spite of this the readings of the thermometer differ so little is owing to the great purity of the liquid hydrogen in the bath.

with the same filling of the thermometer yielded the following results: (cf. Comm, N°. 95^a Tab. VI and N°. 95^c Tab. I) [1])

July 7th, '05	— 139°.867
Oct. 26th, '05	— 139°.873
July 6th, '05	— 217°.416
March 3rd, '06	— 217°.424
June 30th, '06	— 182°.730
July 6th, '06	— 182°.728

For the deviation of the determinations on one day from the mean of the determinations on the two days follows resp.:

0°.003, 0°.004 and 0°.001 so mean 0°.0027,

which harmonizes very well with the mean error derived above for a single observation [2]), from which appears at the same time that different systematic errors are excluded. This justifies at the same time the supposition from which we started, that the error in the resistance thermometer may be neglected.

Determinations with different fillings agree very well.

The determinations made on July 6th, '05 and March 3rd, '06 with the thermometer filled with electrolytic hydrogen (see § 3) and those made on June 30th, '06 with the thermometer filled with distilled hydrogen, give:

mean of July 6th, '05 and March 3rd, '06 — 217°.420

June 30th, '06 5u50′ — 217°.327
6u 5′ — 217°.362
6u25′ — 217°.347
} mean — 217°.345.

If the last temperature is reduced to the same resistance as the first, we find — 217°.400, hence the difference of these values is 0°.020, from which, only one determination being made, we must conclude, that also with regard to the filling systematic errors are pretty well excluded down to — 217°.

§ 8. *Results.*

It appears from the foregoing that with our hydrogen thermometer determinations of temperature, even at the lowest temperatures,

[1]) The temperature for June 30th '06 given here differs slightly from that given in Table I of Comm. N°. 95^c, though both refer to the same resistance. This difference is due to the fact that in Comm. N°. 95^c the result of one reading has been used, and here the mean has been given of more readings.

[2]) It gives namely for the probable error 0°.0029, so only a trifling difference with the above.

may without difficulty be effected accurate to $\frac{1}{50}°$ if the requisite precautions are taken. Though it is not certain that the determinations in liquid hydrogen of the last series come up to this accuracy, as there a systematic error caused by the filling may show its influence, which does not yet make its appearance at —217°, yet it lies to hand to suppose, that, at least with the thermometer filled with distilled hydrogen, also these temperatures may be determined with the same degree of accuracy.

§ 9. *Vapour tension of liquid hydrogen at the melting point.*

By sufficiently lowering the pressure over the bath of liquid hydrogen the temperature was reached at which the hydrogen in the bath becomes solid. This temperature indicates the limit below which accurate determinations are no longer possible by the method discussed in this Communication.

It could be accurately determined by a sudden change in the sound which the valves of the stirrer in the bath bring about. (See Comm. N°. 94*f*, XII § 3).

It appeared from the indication of the resistance thermometer that the gas in the hydrogen thermometer had partly deposited. Hence the pressure in the hydrogen thermometer gives the vapour tension of liquid hydrogen near the melting point. For this we found:

$$H_{Sm} = 53.82 \text{ m.m.}^{1}).$$

§ 10. *Reduction on the absolute scale.*

The reduction of the readings of the hydrogen thermometer on the absolute scale by means of the results of determinations of the isotherms will be discussed in a following Communication.

§ 11. *Variations of the zero point pressure of the thermometer.*

It is noteworthy that the pressure in the thermometer in determinations of the zero point slowly decreases. This change is strongest when the thermometer has just been put together and becomes less in course of time. This is very evident when the results of the determinations made at the beginning of every new period of observation are compared, so after the thermometer has been left unused for some time under excess of pressure.

Thus on the fifth of July '05 shortly after the thermometer had

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[1]) For this Travers, Senter and Jaquerod (loc. cit., p. 170) find a value lying between 49 and 50 m.m. The great difference is probably owing to the inferior accuracy of these last determinations.

been put together we found:

$$H_0 = 1093.10 \text{ mm.}$$

whereas at the beginning of the two following periods of observation was found:

on Oct. 13th '05 $H_0 = 1092.11$ mm.
on Febr. 26th '06 $H_0 = 1091.93$ mm.

The determinations before and after every period of observation give but slight differences when compared. As a rule the pressure decreases slightly as in the second of the above-mentioned periods of observation (March 7th '06, $H_0 = 1091.83$ mm.), sometimes there is a slight increase, as in the first period of observation (Nov. 2nd '05, $H_0 = 1092.23$ mm.) after observations under low pressure. Before and after the last series of observations, when shortly after the thermometer had been filled with distilled hydrogen, determinations were made at $-183°$ and $-217°$, this difference was particularly large. The zero point pressure after the measurements was then 0.33 mm. larger than before them.

From earlier observations made with another thermometer the same thing appeared.

Thus on Nov. 19th '02

$$H_0 = 1056.04 \text{ mm.}$$

was found, and the pressure on June 8th '04 was

$$H_0 = 1055.43 \text{ mm.}$$

while during further measurements up to July 7th '04 the pressure retained a value which within the limits of the errors of observation remained equal to this.

CHAPPUIS[1]) found a similar decrease viz. 0.1 mm. in three months with a zero point pressure of 1 M. of mercury.

Finally a decrease of the normal volume was observed by KUENEN and ROBSON and by KEESOM also with the air manometer (see Comm. N°. 88 (Oct. 1903) III § 3). The same phenomenon was recently observed with the auxiliary manometer filled with hydrogen mentioned in Comm. N°. 78 (March 1902), when it was again compared with the open standard manometer. This comparison will be discussed in a following Communication.

The possibility of there being a leak is excluded by the fact that a final condition is reached with the thermometer.

It lies to hand to attribute the variations of the zero point to an

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[1]) Nouvelles études sur les thermomètres à gaz, Travaux et Mémoires du Bureau International. T. XIII p. 32.

absorption which comforms slowly to the pressure. As to the absorption of the gas in the mercury, its adsorption to the wall and the interchange of gas with a thin layer between the wall and the mercury they (and especially the last) may be left out of account, though they are not rigorously zero. For with manometers, where no influences but these can exert themselves, the pressure of the gas is sometimes considerably raised during a long time, and notwithstanding the variations of the normal volume are much slighter than with the thermometers.

Consequently we shall rather have to think of a slow dissolving in and evaporating from the layer of glue, which is applied between the steel caps and the glass.

ERRATA.

p. 193	l. 1	from top	for: deviation, read: value of the deviations.
	l. 2	from top	for: largest deviation, read: of the largest deviations.
p. 195	l. 8	from bottom	and l. 2 from bottom for: values read: quantities.
p. 196	l. 7	from top	for: from, read: for.
	l. 9	from top	for: and are combined, read: and these are given.
p. 198	l. 19	from top	for: agree, read: correspond.
	l. 6	from bottom	in note, for: calculations, read: calculation of the formulae we used.
	l. 6	from top	must be omitted: "are used"
p. 211	l. 16	from top	for: with, read: containing also.

(November 22, 1906).

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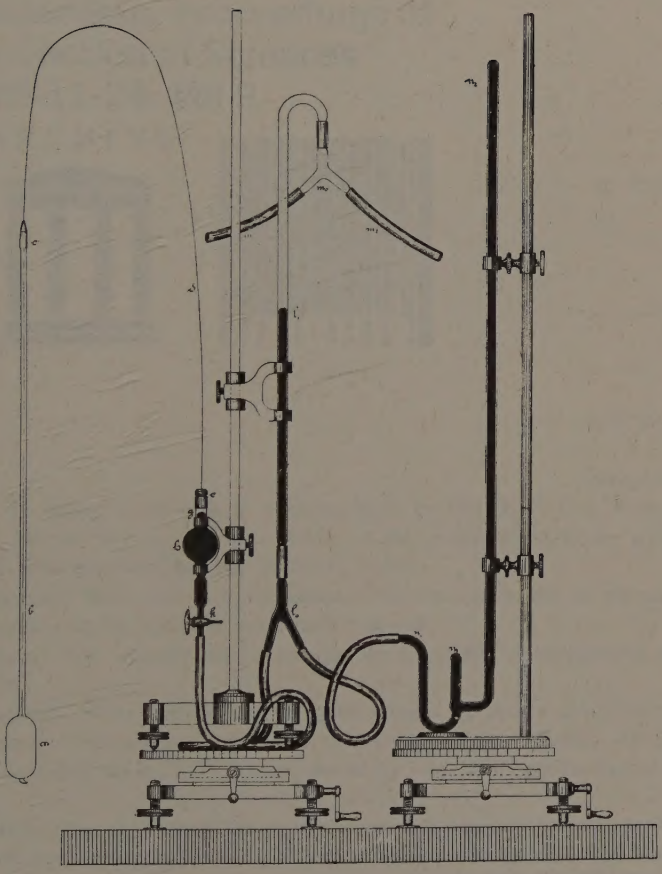
Plate I.